DEPARTMENT OF EDUCATION

DEPARTEMENT VAN ONDERWYS

LEFAPHA LA THUTO

ISEBE LEZEMFUNDO

PROVINSIALE EKSAMEN

PROVINCIAL EXAMINATION

GRAAD 12/GRADE 12

WISKUNDE/MATHEMATICS

VRAESTEL 2/PAPER 2

JUNIE/JUNE 2025

NASIENRIGLYNE/MARKING GUIDELINES

PUNTE/MARKS: 150

TYD/TIME: 3 uur/hours

Hierdie nasienriglyne bestaan uit 17 bladsye./
These marking guidelines consist of 17 pages

Blaai om asseblief/
Please turn over

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

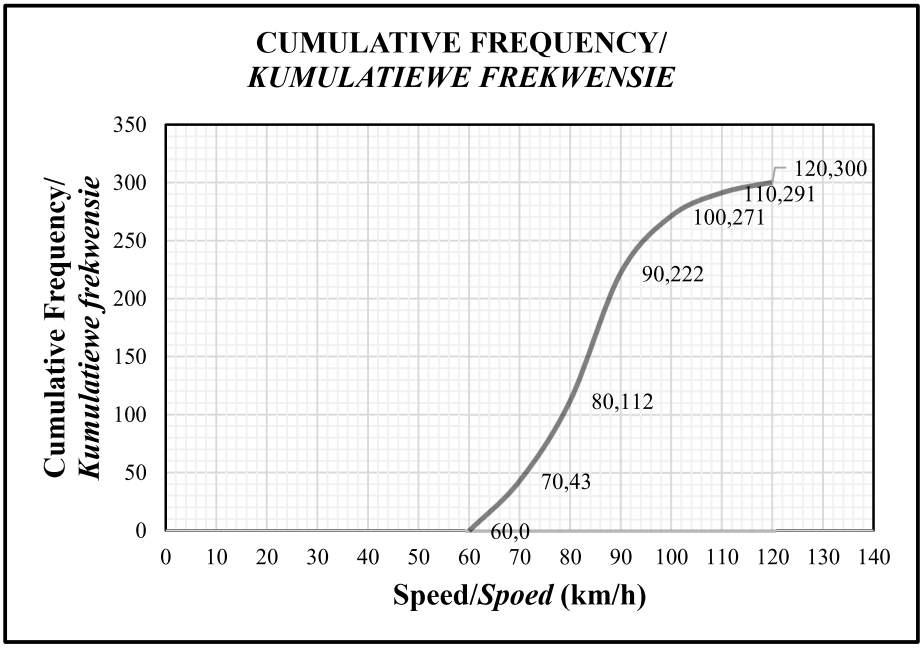
LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/*VRAAG* 1

1.1	$\tilde{x} = 48,45$	✓✓Answer (2)
1.2	$\sigma = 25.97$	✓Answer (1)
1.3	[48,45 – 25,97 ; 48,45 + 25,97] [22,48 ; 74,42] 8 learners	✓22.48 ✓74.42 ✓8 learners (3)
1.4	IQR=55-31 = 24	✓55 ✓31 ✓Answer (3)
1.5	112 $Q_3 + 1.5.IQR = 55 + 1.5(24) = 91$ $112 > 91$	✓Answer (1)
		[10]

QUESTION/*VRAAG*2

2.1	43 + 69 + 110 + 49 + 20 + 9 = 300 cars	✓300 (1)
2.2		✓43,112, 222 ✓271,291, 300 (2)
2.3		✓ grounding ✓Upper boundaries . ✓shape (3)
2.4	$Q_1 = 75$ $Q_3 = 90$ accept Q1 from 72-75, accept Q3 from 90-93 $SIQR = \frac{90-75}{2} = 7.5$ CA from Ogive	✓75 ✓90 ✓Answer (3)
2.5	$300 - 212 = 188\,cars$ (If read from ogive-accept from 187-189) 300- 112 Could also just add using the table. CA from learners' ogive all the way	✓✓188 (2)
		[11]

Speed/*Spoed* km/h	Frequency/ *Frekwensie*	F
$60 \le x < 70$	43	43
$70 \le x < 80$	69	112
$80 \le x < 90$	110	222
$90 \le x < 100$	49	271
$100 \le x < 110$	20	291
$110 \le x < 120$	9	300

QUESTION/*VRAAG* 3

3.1	$m=\frac{6-0}{3-0}$ Not allowed to use the value of S in q 3.1 - 3.3 $=2$		✓Substitution ✓Answer (2)
3.2	$\tan\theta=2$ $\theta=\tan^{-1}(2)$ $=63.43^{\circ}$		✓Substitution ✓Answer (2)
3.3.1	$\alpha=63.43^{\circ}+20.23^{\circ}\ \ (ext\angle of\ \Delta)$ $\alpha=83.66^{\circ}$ $m=\tan 83.66^{\circ}$ $m=9$		✓ $\alpha=83.66^{\circ}$ ✓substitution ✓Answer (3)
3.3.2	$y=9x+c$ $3=9(-2)+c$ $3+18=c$ $c=21$ $y=9x+21$		✓✓substitution ✓Answer (3)
3.4	$2x=9x+21$ Mistake in question paper. S given in paper differs from S calculated in 3.4 $-7x=21$ $x=-3$ $y=2(-3)$ $y=-6$ $S(-3;-6)$		✓equating ✓ $x=-3$ ✓substitution ✓ $y=-6$ (4)
3.5	$S(-21;-42)$ $M\left(\frac{-21+3}{2};\frac{6+(-42)}{2}\right)$ $M(-9;-18)$	$S(-3;-6)$ $M\left(\frac{-3+3}{2};\frac{-6+6}{2}\right)$ $M(0;0)$	✓ x-value ✓ y-value (2)

3.6	$-9=\frac{-2+x}{2}$ $-18=\frac{3+y}{2}$ $-18+2=x$ $-36-3=y$ $R(-16;-39)$	$0=\frac{-2+x}{2}$ $x=2$ $0=\frac{3+y}{2}$ $y=-3$ $R(2;-3)$ Answer only: full marks	✓substitution ✓x-value ✓y-value (3)
			[19]

QUESTION/*VRAAG* 4

4.1.1	$0=\frac{-5}{2}x-4$ $4=\frac{-5}{2}x$ $8=-5x$ $x=-\frac{8}{5}$ $E\left(\frac{-8}{5};0\right)$	✓0 ✓$-\frac{8}{5}$ (2)
4.1.2	$x^2+y^2=r^2$ $(0)^2+\left(-\frac{8}{5}\right)^2=r^2$ $r^2=\frac{64}{25}$ $x^2+y^2=\frac{64}{25}$ NB It was not given that AMB is a diameter. BUT the sketch is misleading. The option is therefor added to the memo, BUT it is actualluy INCORRECT to assume that AMB is a diameter.	✓substitution ✓$r^2=\frac{64}{25}$ ✓answer (3)
4.2	CORRECT $M_{AE}=-\frac{5}{2}$ $M_{AM}=\frac{2}{5}\left(\tan\perp radii\right)$ $y=\frac{2}{5}x+c$ $c=-4$ $y=\frac{2}{5}x-4$ $M_{AM}=\frac{-2-(-4)}{5-0}$ $=\frac{2}{5}$ $c=-4$ $y=\frac{2}{5}x-4$ xx ASSUMED	✓gradient of AM ✓$y=\frac{2}{5}x+c$ ✓Answer (3)
4.3	$-3=\frac{2}{5}a-4$ CORRECT $1=\frac{2}{5}a$ $a=\frac{5}{2}$ OR $a=\frac{5+0}{2}$ xx ASSUMED $a=\frac{5}{2}$	✓substitution ✓simplification ✓Answer (3) OR ✓✓substitution ✓Answer (3)

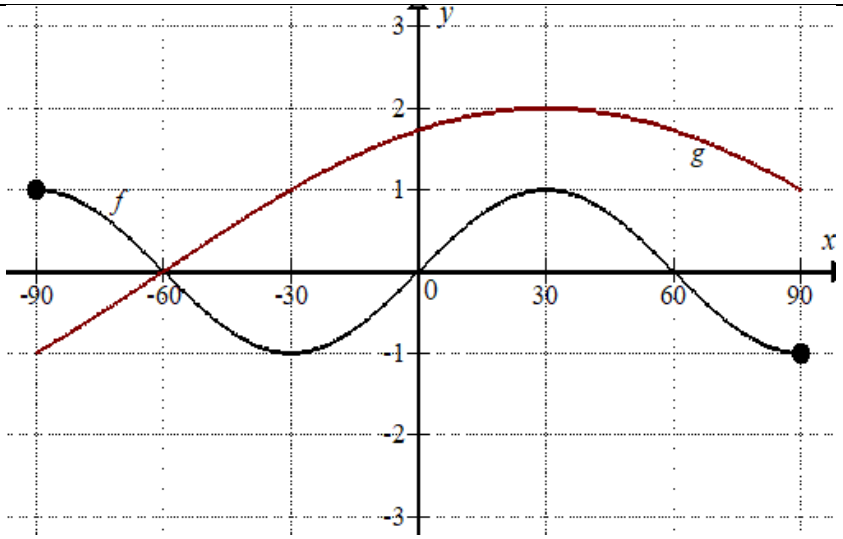
4.4	$\left(x-\frac{5}{2}\right)^2+\left(y-(-3)\right)^2=r^2$ $\left(x-\frac{5}{2}\right)^2+(y+3)^2=r^2$ $\left(0-\frac{5}{2}\right)^2+(-4+3)^2=r^2$ $r^2=\frac{29}{4}$ $\left(x-\frac{5}{2}\right)^2+(y+3)^2=\frac{29}{4}$ AO 1 out of 4	✓$\left(x-\frac{5}{2}\right)^2+(y+3)^2=r^2$ ✓Substitution ✓$r^2=\frac{29}{4}$ ✓Answer (4)
4.5	$\hat{G}=90^\circ\ (\angle\ in\ semi-circle)$ $G\left(0;\frac{8}{5}\right)$ $F\left(\frac{8}{5};0\right)$ $EG=\sqrt{\left(\frac{-8}{5}-0\right)^2+\left(\frac{8}{5}-0\right)^2}=\frac{8\sqrt{2}}{5}$ $GF=\frac{8\sqrt{2}}{5}$ Area of $\Delta EFG=\frac{1}{2}\left(\frac{8\sqrt{2}}{5}\right)\left(\frac{8\sqrt{2}}{5}\right)$ $=\frac{64}{25}=2.56\ units^2$ OR Area of $\Delta EFG=\frac{1}{2}.EF.OG$ Area of $\Delta EFG=\frac{1}{2}\left(2.\frac{8}{5}\right)\left(\frac{8}{5}\right)$ $=\frac{64}{25}=2.56\ units^2$	✓Angle G. ✓length of EG ✓length of GF ✓Substitution ✓Answer (5) OR ✓✓method ✓✓substitution ✓Answer (5)
		[20]

QUESTION/*VRAAG* 5

5.1.1	$\left(\sqrt{34}\right)^2 = (3)^2 + p^2$ $34 - 9 = p^2$ $p = -5$	✓substitution ✓Answer (2)
5.1.2	$\cos\left(450^\circ - 2\theta\right)$ $= \sin 2\theta$ $= 2\sin\theta.\cos\theta$ First mark awarded here $= 2\left(\frac{-5}{\sqrt{34}}\right)\left(\frac{3}{\sqrt{34}}\right)$ $= \frac{-30}{34}$ $= \frac{-15}{17}$	✓ $\sin 2\theta = 2\sin\theta\cos\theta$ ✓substitution ✓Answer (3)
5.1.3	$\cos\left(30^\circ - \theta\right)$ $\cos 30^\circ.\cos\theta + \sin 30^\circ.\sin\theta$ $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3}{\sqrt{34}}\right) + \left(\frac{1}{2}\right)\left(\frac{-5}{\sqrt{34}}\right)$ $\frac{3\sqrt{3} - 5}{\sqrt{34}}$ Answer $\frac{3\sqrt{102} - 5\sqrt{34}}{34}$	✓Expansion ✓Substitution ✓Answer (3)
5.2	$\frac{-2\sin x.-\cos x}{\sin\left(x + 60^\circ + x\right)}$ Question paper misleading...does not simplify to a single trigonometric function $\frac{2\sin x.\cos}{\sin\left(2x + 60^\circ\right)}$ $\frac{\sin 2x}{\sin 2x.\cos 60^\circ + \cos 2x.\sin 60^\circ}$ $\frac{\sin 2x}{\frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x}$ $\frac{2\sin 2x}{\sin 2x + \sqrt{3}\cos 2x}$ $\frac{2\tan 2x}{\tan 2x + \sqrt{3}}$	✓ $-\sin x$ ✓ $-\cos x$ ✓compound angle ✓ $\sin 2x$ ✓ $\sin 2x.\cos 60^\circ + \cos 2x.\sin 60^\circ$ ✓Answer (6)

5.3.1	$f(x)=\cos(x+45^\circ).\cos(45^\circ-x)$ $=\left[\cos x.\cos 45^\circ-\sin x.\sin 45^\circ\right]\left[\cos x.\cos 45^\circ+\sin x.\sin 45^\circ\right]$ $=\left[\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right]\left[\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right]$ $=\frac{1}{2}\cos^2 x-\frac{1}{2}\sin^2 x$ $=\frac{1}{2}\left(\cos^2 x-\sin^2 x\right)$ $=\frac{1}{2}\cos 2x$	✓first expansion ✓second expansion ✓$\frac{1}{2}\cos^2 x-\frac{1}{2}\sin^2 x$ ✓common factor (4)
5.3.2	$\frac{1}{2}\cos 2x=1-2\sin x$ $1-2\sin^2 x=2-4\sin x$ $2\sin^2 x-4\sin x+1=0$ $\sin x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(1)}}{2(2)}$ $\sin x=\frac{2-\sqrt{2}}{2}=0,29 \quad or \quad \sin x=\frac{2+\sqrt{2}}{2}=1,71$ $x=17.03^\circ+k.360^\circ \qquad no\ solution$ $x=162.97^\circ+k.360^\circ \qquad k\in Z$ CA is learner uses 0,293 etc	✓formulae ✓standard form ✓$\sin x$ value(s) ✓$x=17.03^\circ+k.360^\circ$ ✓$x=162.97^\circ+k.360^\circ$ ✓$k\in Z$ (6)
5.4	$\cos\theta=2m \quad and \quad \cos 2\theta=7m$ $2\cos^2\theta-1=7m$ $2(2m)^2-1=7m$ $8m^2-7m-1=0$ $(8m+1)(m-1)=0$ $m=\frac{-1}{8} \quad or \quad m=1$ $\therefore m=-\frac{1}{8}$	✓formulae ✓substitution ✓standard form ✓factors ✓Answer with selection (5)
		[29]

QUESTION/*VRAAG* 6

6.1	120°	✓Answer (1)
6.2		*g* ✓shape ✓*x*-intercept ✓turning point (3)
6.3.1	$x \in [-90^\circ; -60^\circ)$ *or* $-90^\circ \leq x < -60^\circ$	✓✓Answer (2)
6.3.2	$-90^\circ < x < 0^\circ$ *or* $60^\circ < x < 90^\circ$, $x \neq -60^\circ$ Incorrect , some equal signs omitted	✓ $-90^\circ < x < 0^\circ$ ✓ $60^\circ < x < 90^\circ$ ✓ $x \neq -60^\circ$ (3)
6.3.3	$x = 30^\circ$ Question paper asked wrong way round - the only answer available is a -1. Therefor...actually no solution **Note: Accept no solution**	✓Answer (1)
6.4	$h(x) = 2\cos(x - 30^\circ - 60^\circ)$ $= 2\cos(x - 90^\circ)$ $= 2\sin x$	✓ $2\cos(x - 90^\circ)$ ✓Answer (2)
		[12]

QUESTION/*VRAAG* 7

7.1	in ΔPQS $\cos 30^\circ = \frac{x}{PS}$ $\frac{\sqrt{3}}{2} = \frac{x}{PS}$ $PS = \frac{2x}{\sqrt{3}}$ in ΔPQR $\sin 30^\circ = \frac{x}{PR}$ $\frac{1}{2} = \frac{x}{PR}$ $PR = 2x$ in ΔPSR $PR^2 = RS^2 + PS^2 - 2RS.PS\cos\theta$ $(2x)^2 = \left(\frac{2x}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 - 2\left(\frac{2x}{\sqrt{3}}\right)\left(\sqrt{3}\right)\cos\theta$ $4x^2 = \frac{4}{3}x^2 + 3 - 4x\cos\theta \quad \times 3$ $12x^2 = 4x^2 + 9 - 12x\cos\theta$ $12x\cos\theta = 9 - 8x^2$ $\cos\theta = \frac{9-8x^2}{12x}$	✓ $\cos 30^\circ = \frac{x}{PS}$ ✓ $PS = \frac{2x}{\sqrt{3}}$ ✓ $\sin 30^\circ = \frac{x}{PR}$ ✓ $PR = 2x$ ✓Substitution into Cosine rule ✓ $4x^2 = \frac{4}{3}x^2 + 3 - 4x\cos\theta$ (6)
7.2	Area of $\Delta PSR = \frac{1}{2}\left(\sqrt{3}\right)\left(\frac{2(1)}{\sqrt{3}}\right)\sin\theta$ $= \sin\theta$ units^2	✓✓Substitution into Area rule (2)
		[8]

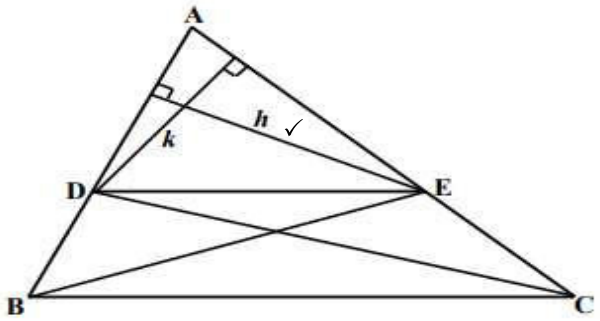
QUESTION/*VRAAG* 8

8.1	$\hat{QOR} = 228^\circ \ (\angle \ around \ a \ point)$ $\hat{QTR} = 114^\circ \ (\angle \ at \ center = 2\angle \ at \ circumference)$ OR $\hat{RPQ} = 66^\circ \ (\angle \ at \ center = 2\angle \ at \ circumference)$ $\hat{QTR} = 114^\circ \ (opp \ \angle s \ of \ cyclic \ quad)$	✓S ✓S ✓R OR ✓S✓R ✓S (3)
8.2	$\hat{OQS} = 90^\circ \ (\tan \perp radii)$ $\hat{ORS} = 90^\circ \ (\tan \perp radii)$ $\therefore$ QORS is a cyclic quadrilateral $(opp \ \angle s$ are supplementary$)$ OR (converse of opp $\angle s$ of cyclic quad)	✓S/R ✓S/R ✓R (3)
8.3	$\hat{QSR} = 48^\circ$ (Opp $\angle s$ of cyclic quad) OR (sum of $\angle s$ of quad)	✓S ✓R (2)
		[8]

QUESTION/*VRAAG* 9

9.1	$c = x$ (tan–*chord theorem*) S R $E_1 = 180^\circ - (x+y)$ (*sum of* $\angle s$ *in* Δ) S/R $E_3 = y$ ($\angle s$ *in str line*) / *ext* $\angle$ *of* ΔDEC S/R $A\hat{B}E = y$ ($\angle s$ *opp* = *sides*) S/R $D_1 = A\hat{B}E = y$ $\therefore$ ABED is a cylic quad (converse of ext $\angle$ of cyclic quad) R	✓S✓R ✓S/R ✓S/R ✓S/R ✓R (6)
9.2	$\hat{B}_1 = x$ ($\angle s$ *in same seg*) S R $\hat{B}_1 = \hat{C} = x$ AB is a tangent to a circle B,C and D (converse of tanchord theorem) *or* ($\angle$ between a line and chord) R	✓S ✓R ✓R (3)
		[9]

QUESTION/*VRAAG* 10

10.1	 Construct height h and k perpendicular to AD and AE respectively. Join DC and BE. $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2}AD.h}{\frac{1}{2}.BD.h}$ ✓ $(\text{Same height } h)$ $= \frac{AD}{BD}$ $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED} = \frac{\frac{1}{2}AE.k}{\frac{1}{2}.EC.k}$ ✓ $(\text{Same height } k)$ $= \frac{AE}{EC}$ But Area of ΔBDE=Area of ΔCED ✓ ✓ $(\text{same base DE,same height between} \parallel \text{lines.})$ $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED}$ ✓ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓construction ✓S ✓S ✓S ✓R ✓S (6)

10.2	$\frac{AD}{DB}=\frac{AF}{FC}\ (\text{Prop theorem } DF\|BC)$ $AF=\frac{AD.FC}{DB}$ $From\ \frac{AF}{FE}=\frac{FC}{EB}$ $AF=\frac{FE.FC}{EB}$ $\frac{FE.FC}{EB}=\frac{AD.FC}{DB}$ $\frac{FE}{EB}=\frac{AD}{DB}$ $\therefore DE\|AF\ (\text{Converse of prop theorem})$	✓S ✓R ✓ $AF=\frac{AD.FC}{DB}$ ✓ $AF=\frac{FE.FC}{EB}$ ✓Equating ✓R (6)
		[12]

QUESTION/*VRAAG* 11 The word "TANGENT" DE was omitted on ENGLISH side

11.1	$F\hat{G}E = \theta\ (Tan-chord\ theorem)$ $\hat{E}_2 = \theta\ (\angle s\ opp = sides)$ $\hat{F}_2 = \theta (Alt\ \angle s\ , DE \| FH)$	✓S ✓S ✓S (3)
11.2	In ΔDEF and ΔDGE ✓ $\hat{D} = \hat{D}$ common ✓ $\hat{E}_1 = \hat{G}$ tan chord theorem or (proved in 11.1) ✓ $\hat{F}_3 = \hat{E}_1 + \hat{E}_2$ (Sum of $\angle s$ in Δ) ΔDEF /// ΔDGE (AAA) ✓ $\frac{DE}{DG} = \frac{EF}{GE} = \frac{DF}{DE}$ ✓ (from /// Δs) $DE^2 = DF.DG$	✓identifying triangles ✓S/R ✓S ✓R(AAA) ✓S (5)
11.3	$\frac{DF^2}{DE^2} + \frac{DF}{DE} = \frac{DF^2 + DF.DE}{DE^2}$ $= \frac{DF(DF+DE)}{DE^2}$ $= \frac{DF(DF+FG)}{DF.DG}$ $(DE = FG)$ $= \frac{DF+FG}{DG}$ $(DE^2 = DF.DG)$ $= \frac{DG}{DG}$ $(DG = DF.FG)$ $= 1$ OR $\frac{EF}{GE} = DE^2 = DF.DG$ $= DF.(DF + FG)$ $= DF^2 + DF.FG$ $= DF^2 + DF.DE$ $\frac{DE^2}{DE^2} = \frac{DF^2}{DE^2} + \frac{DF.DE}{DE^2}$ $1 = \frac{DF^2}{DE^2} + \frac{DF}{DE}$	✓ $\frac{DF^2 + DF.DE}{DE^2}$ ✓common factor ✓S ✓S (4) ✓ $DF.(DF + FG)$ ✓ $DF^2 + DF.FG$ ✓ $DF^2 + DF.DE$ ✓Dividing both sides by DE^2 (4) **[12]**
		TOTAL: 150