education

Department of Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P2

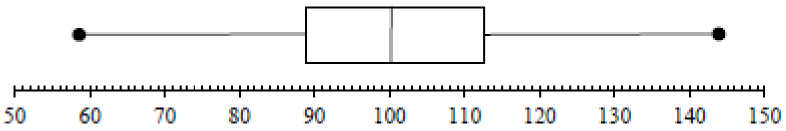
SEPTEMBER 2018

MARKS: 150

MARKING GUIDELINE

This marking guideline consists of 14 pages.

QUESTION 1

1.1	Lower quartile : $Q_1 = 89$ Upper quartile : $Q_3 = 113$	✓ 89 ✓ 113 (2)
1.2		✓ Min = 58 and max = 145 ✓ $Q_1 = 89$ and $Q_3 = 113$ ✓ $Q_2 = 100$ (3)
1.3	Symmetrical accept both Marginally skewed to the right	✓ Answer (1)
1.4	Mean : $\bar{x} = \frac{1522}{15}$ $\bar{x} = 101{,}47$	✓✓ $\frac{1522}{15}$ (2)
1.5	Standard deviation : $\sigma = 19{,}07$	✓ $\sigma = 19{,}07$ (1)
1.6	$(\bar{x} - \sigma ; \bar{x} + \sigma)$ $= (82{,}4 ; 120{,}54)$ 2 days	✓ 82,4 ✓ 120,54 ✓ 2 days (3)
		[12]

QUESTION 2

2.1	$r = -0{,}95$	✓ ✓ value of r (2)
2.2	There is a strong negative relationship.	✓strong negative (1)
2.3	$a = 11{,}71323529$ $b = -1{,}11764$ $\hat{y} = 11{,}71 - 1{,}12x$	✓ $a = 11{,}71$ ✓ $b = -1{,}12$ ✓ correct equation (3)
2.4	$\hat{y} = 11{,}71 - 1{,}12(6)$ $\hat{y} = 4{,}99$ $\hat{y} \approx 5$ times **OR** $\hat{y} = 5{,}007$ $= 5$ times	✓ Substitution ✓ $y \approx 5$ times ✓ calculator ✓ $\hat{y} \approx 5$ times (2)
		[8]

QUESTION 3

3.1.1	$M_{EF} = M_{FG}$ $\frac{-1-3}{0-4} = \frac{1-(-1)}{t}$ $\frac{-4}{-4} = \frac{1+1}{t}$ $1 = \frac{2}{t}$ $t = 2$	✓ $M_{EF} = M_{FG}$ ✓ Substitution into the correct formula ✓ $1 = \frac{2}{t}$ ✓ Answer (4)
3.1.2	$M_{EF} \times M_{FG} = -1$ $1 \times \frac{2}{t} = -1$ $t = -2$	✓ $1 \times \frac{2}{t} = -1$ ✓ Answer (2)

3.2.1	$y=\frac{1}{2}x+8$ $M_{QR}=M_{PS}=\frac{1}{2}$ QR//PS $y-5=\frac{1}{2}(x-4)$ $y=\frac{1}{2}x-2+5$ $\therefore y=\frac{1}{2}x+3$	✓ rewrite equation of PR ✓ $M_{PS}=\frac{1}{2}$ ✓ Substitute $(4;5)$ correctly ✓ Equation (4)
3.2.2	$0=\frac{1}{2}x+3$ $x=-6$ $\therefore S(-6;0)$	✓ $y=0$ ✓ $S(-6;0)$ (2)
3.2.3	$\text{M}\left(\frac{2+(-6)}{2};\frac{6+0}{2}\right)$ $=\text{M}\,(-2;3)$	✓ $\frac{2+(-6)}{2}$ ✓ $\frac{6+0}{2}$ (2)
3.2.4	Midpoint of QS = midpoint of PR (diagonals bisect) $-2=\frac{x+4}{2}$ $x=-8$ $3=\frac{y+5}{2}$ $y=1$ $\therefore \text{R}(-8;1)$ **OR** $\text{P}\to Q:(x-2;y+1)$ $\text{S}\to R:(x-2:y+1)$ $\therefore R(-6-2:0+1)$ $\therefore R(-8:1)$	✓ $-2=\frac{x+4}{2}$ ✓ $3=\frac{y+5}{2}$ ✓ $\text{R}(-8;1)$ (3) ✓ $\to R:(x-2:y+1)$ ✓✓ $\therefore R(-8:1)$ (3)
3.2.5	$m_{PS}=\frac{1}{2}$ $PS\,//\,QR$ $\tan O\hat{S}P=m_{PS}=\frac{1}{2}$ $\therefore O\hat{S}P=26{,}57°$ $m_{RS}=-\frac{1}{2}$ $R\hat{S}\text{O}=180°-26{,}57°$ $=153{,}43°$ $\therefore R\hat{S}P=153{,}43°-26{,}57°$ $=126{,}86°$	✓ $\tan O\hat{S}P=m_{PS}=\frac{1}{2}$ ✓ $O\hat{S}P=26{,}57°$ ✓ $m_{RS}=-\frac{1}{2}$ ✓ $R\hat{S}\text{O}=153{,}43°$ ✓ $126{,}86°$ (5)

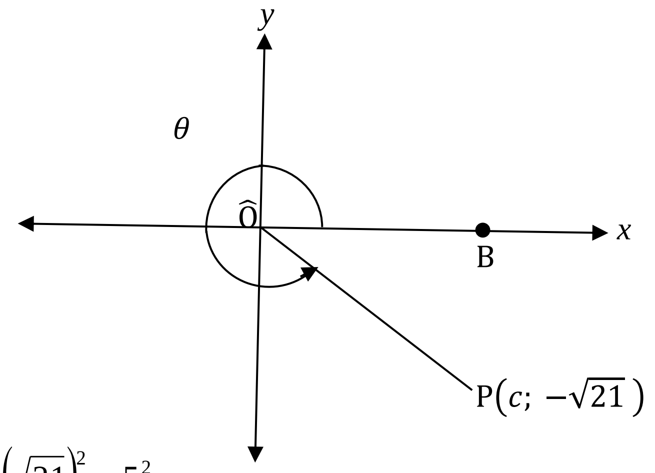
		[22]

QUESTION 4

4.1	$M\left(\frac{-2+4}{2};\frac{3+(-7)}{2}\right)$ $=M(1;-2)$ $\therefore$ centre of the circle is $(1;-2)$ $(x-1)^2+(y-(-2))^2=r^2$ $(-2-1)^2+(3+2)^2=r^2$ $9+25=r^2$ $\therefore 34=r^2$ $(x-1)^2+(y+2)^2=34$	✓ Subst. into the correct formula ✓ Centre of the circle ✓ Subst. $N(-2;3)$ ✓ $34=r^2$ ✓ Equa of the circle (5)
4.2	$M_{rad}=\frac{-2-3}{1-(-2)}$ $=\frac{-5}{3}$ $M_{\tan}=\frac{3}{5}$ radius $\perp$ tangent $y-3=\frac{3}{5}(x-(-2))$ $\therefore y=\frac{3}{5}x+\frac{6}{5}+3$ $=\frac{3}{5}x+\frac{21}{5}$	✓ Subst. into the correct formula ✓ $M_{rad}=\frac{-5}{3}$ ✓ $M_{\tan}=\frac{3}{5}$ ✓ Subst. $N(-2;3)$ and gradient ✓ Equation (5)

4.3	Let the angle of inclination of line NP be α and the angle adjacent to it be θ $\tan\alpha = \frac{-5}{3}$ $\alpha = 180° - \tan\left(\frac{5}{3}\right)$ $\therefore \alpha = 180° - 59{,}036243...°$ $= 120{,}96°$ $\theta = 59{,}04°$ $N\hat{P}Q = 180° - (59{,}04° + 90°)$ $= 30{,}96°$ **OR** $N\hat{Q}P = 120{,}96° - 90°$ $= 30{,}96°$ (ext.$<$ of Δ) **OR** $N\hat{Q}P = 90°$ diameter subtends a right angle $NQ = 6$ and $PQ = 10$ $\therefore \tan N\hat{P}Q = \frac{6}{10}$ $\therefore N\hat{Q}P = 30{,}96°$	✓ $\tan\alpha = -\frac{5}{3}$ ✓ $\alpha = 120{,}96°$ ✓ $\theta = 59{,}04°$ ✓ $N\hat{P}Q = 30{,}96°$ (4) ✓ $N\hat{Q}P = 90°$ ✓NQ = 6 and PQ = 10 ✓ $\tan N\hat{P}Q = \frac{6}{10}$ ✓ $N\hat{Q}P = 30{,}96°$ (4)
4.4	QP = 10 units and NQ = 6 units Area ΔNPQ = $\frac{1}{2}(6)(10)$ ✓ = 30 square units $\therefore \frac{\text{Area } \Delta NPQ}{\text{Area } of\ circle} = \frac{30✓}{34\pi ✓}$ = 0,28 ✓	(4)
		[17]

QUESTION 5

5.1.1	Reflex $\mathrm{B\hat{O}P} = \theta$  $c^2 + \left(\sqrt{21}\right)^2 = 5^2$ $c^2 = 25 - 21$ $c^2 = 4$ $\therefore c = 2$	✓ subst into pyth ✓ Answer (2)
5.1.2	a) $\cos\theta = \frac{2}{5}$	✓ Answer (1)
	b) $= \frac{-\sqrt{21}}{2} + \left(\frac{-\sqrt{21}}{5}\right)^2$ $= \frac{-25\sqrt{21} + 42}{50}$	✓ $\frac{\sqrt{21}}{2} + \left(\frac{\sqrt{21}}{5}\right)^2$ for corr subst ✓ Answer (2)
	c) $2\sin\theta\cos\theta$ $= 2\left(\frac{-\sqrt{21}}{5}\right)\left(\frac{2}{5}\right)$ $= \frac{-4\sqrt{21}}{25}$	✓ Identity ✓ Answer (2)
5.2	$\frac{(-\sin x).\tan x.\cos(360° - 30°)}{(\sin x)^2}$ $= \frac{-\sin\ x.\tan x(\cos 30°)}{\sin^2 x.}$ $= \frac{-\sqrt{3}}{2}\tan x \div \sin x$ $= \frac{-\sqrt{3}\sin x}{2\cos x}$ X $\frac{1}{\sin x}$ $= \frac{-\sqrt{3}}{2\cos x}$	✓ $-\sin x$ ✓ $(\sin x)^2$ ✓ $\cos 30°$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer (5)
		[12]

QUESTION 6

6.1	$LHS = \dfrac{2\sin^2 x}{2\dfrac{\sin x}{\cos x} - 2\sin x\cos x}$ $= \dfrac{2\sin^2 x}{2\sin x\left(\dfrac{1}{\cos x} - \cos x\right)}$ $= \dfrac{\sin x}{\dfrac{1-\cos^2 x}{\cos x}}$ $= \sin x \times \dfrac{\cos x}{\sin^2 x}$ $= \dfrac{\cos x}{\sin x}$ $\therefore$ LHS = RHS	✓ $\dfrac{\sin x}{\cos x}$ ✓ $2sin\ xcos\ x$ ✓ $2\sin x\left(\frac{1}{\cos x} - \cos x\right)$ ✓ $\dfrac{\sin x}{\dfrac{1-\cos^2 x}{\cos x}}$ ✓ sin^2x (5)
6.2	This identity will be undefined when $2 + 2\cos 2x = 0$ $\cos 2x = -1$ $\therefore 2x = 180° + k.360°\ \ k\epsilon\mathbb{Z}$ $x = 90° + k.180°$ NB: Penalise 1 mark for leaving out $k\epsilon\mathbb{Z}$ **OR** $2 + 2\cos 2x = 0$ $1 + \cos 2x = 0$ $1 + 2cos^2x - 1 = 0$ $\cos x = 0$ $\therefore x = 90°$ $x = \pm 90° + 360°k; k \in \mathbb{Z}$	✓ $\cos 2x = -1$ ✓ $2x = 180° + k.360°; k\epsilon\mathbb{Z}$ ✓ $x = 90° +$ ✓$k.180°, ; k\epsilon\mathbb{Z}$ (4) ✓ Identity ✓ $\cos x = 0$ ✓ $\pm 90° +$ ✓k $360°; k \in \mathbb{Z}$ (4)
		[9]

QUESTION 7

7.1	$D\hat{E}\,F = 180° - \theta$ opp angle of cyclic quad	✓ Answer (1)
7.2	$DF = \sqrt{r^2 + r^2 - 2r^2\cos(180° - \theta)}$ $= \sqrt{2r^2 + 2r^2\cos\theta}$ $= r\sqrt{2 + 2\cos\theta}$ given	✓ correct sub into cosine *rule* ✓ simplification (2)
7.3	$D\hat{F}G = 90°$ angl in semi circle In ΔDFG: $\sin\theta = \frac{DF}{DG}$ $\sin\theta = \frac{r\sqrt{2+2\cos\theta}}{2r}$ $(2\sin\theta)^2 = (\sqrt{2+2\cos\theta})^2$ $4\sin^2\theta = 2 + 2\cos\theta$ $2\sin^2\theta = 1 + \cos\theta$	✓$\sin\theta = \frac{r\sqrt{2+2\cos\theta}}{2r}$ ✓ $4\sin^2\theta = 2 + 2\cos\theta$ ✓$2\sin^2\theta = 1 + \cos\theta$ (3)
7.4	$2(1 - \cos^2\theta) = 1 + \cos\theta$ $2 - 2\cos^2\theta = 1 + \cos\theta$ $-2\cos^2\theta - \cos\theta + 1 = 0$ $2\cos^2\theta + \cos\theta - 1 = 0$ $(2\cos\theta - 1)(\cos\theta + 1) = 0$ $\cos\theta = \frac{1}{2}$ or $\cos\theta \neq -1$ $\therefore \theta = 60°$ $\theta \neq 180°$	✓ $2(1 - \cos^2\theta) = 1 + \cos\theta$ ✓ standard form ✓ factors ✓ both equations ✓ $\theta = 60°$ (5)
		[11]

QUESTION 8

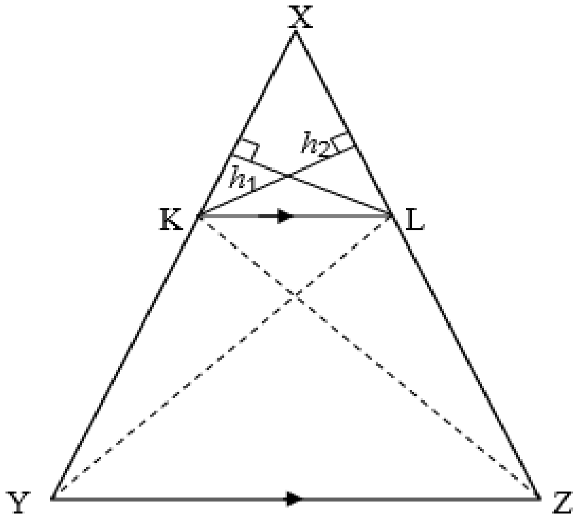
8.1	$a = 1$ $b = 2$ $c = 2$ $d = 1$	✓ $a = 1$ ✓ $b = 2$ ✓ $c = 2$ ✓ $d = 1$ (4)
8.2	$P(21{,}44°;\, 0{,}73)$	✓ correct substitution (2)
8.3.1	$x = 90°$	✓ 90° (1)
8.3.2	$x \in [45°;135°]$ OR $45° \leq x \leq 135°$	✓ $45°$ *and* $135°$ ✓ Notation (2)
		[9]

QUESTION 9

9.1.1	$\hat{C} = \hat{A}_1 = 78°$ alt <'s AP // CB	✓ R (1)
9.1.2	$A\hat{B}C = \hat{A}_1 = 78°$ tan chord theorem	✓ S✓R (2)
9.1.3	$\hat{A}_3 = \hat{C} = 78°$ tan chord theorem	✓ S (1)
9.1.4	$\hat{A}_1 + \hat{A}_2 + \hat{A}_3 = 180°$ straight line $78° + \hat{A}_2 + 18° = 180°$ $\hat{A}_2 = 24°$	✓ S (1)

9.2.1	$\hat{Y}_1 = a$ tan chord theorem $\hat{Y}_1 = Y\hat{X}P = a$ alt angle XY // QT $Y\hat{X}P = \hat{Q} = a$ $=$ angles opp $= sides$ $\hat{Y}_2 = \hat{Q} = a$ angles in same circle segment	✓S✓R✓ ✓S ✓S ✓S✓R✓ (6)
9.2.2	$\hat{T}_2 = X\hat{Y}T$ alt angles; XY // QT $= \hat{Y}_1 + \hat{Y}_2$ $= \hat{T}_1 + \hat{T}_1$ $= 2\hat{T}_1$	✓ S/R ✓ (2)
9.2.3	$\hat{T}_2 + \hat{T}_1 = 90°$ angle in $\frac{1}{2}\odot$ $\hat{T}_2 = 90° - a$	S✓R✓ (2)
9.2.4	$\hat{T}_2 = 90° - a$ proven But $Y\hat{O}Q = 2\hat{T}_2$ angle at centre $=$ 2 angl on circle $= 2(90° - a)$ $= 180° - 2a$ $Q\hat{O}R = 180° - 180° + 2a$ $= 2a$ $\therefore$ SORT is a cyclic quad ext angl $=$ opp int angle **OR** $X\hat{Y}O = 90°$ tan$\perp$radius $X\hat{O}Y = 90° - \propto$ sum of $< s$ of Δ $X\hat{O}Y = \hat{T}_2$ $\therefore$ SORT is a cyclic quad/converse: ext. <s of cyclic quad **OR** $X\hat{O}Y = 2\hat{T}_1$ $<$ at centre $= 2 <$ at circum. $X\hat{O}Y = \hat{T}_2$ $\therefore$ SORT is a cyclic quad/converse: ext. $<$ of cyclic quad	✓ S ✓ R ✓S ✓R ✓S ✓R (2)
9.2.5	$\hat{T}_1 + \hat{T}_2 = 90°$ $a + 2a = 90°$ $3a = 90°$ $\therefore a = 30°$	✓ S ✓ answer (2)
9.2.6	$X\hat{Y}O = 90°$ rad $\perp$tangent $\hat{R}_2 = 90°$ alt.angles; XY // TQ $\therefore TR = RQ$ line from centre of circle$\perp$ chord	✓ S ✓ S ✓ R (3)
		[22]

QUESTION 10



10.1.1	Const: Join KZ & LY & draw h_1 from K ⊥ XL & h_2 from L ⊥ XK Proof $\frac{Area\ \Delta XKL}{Area\ \Delta LYK} = \frac{\frac{1}{2}XK \times h_1}{\frac{1}{2}KY \times h_1} = \frac{XK}{KY}$ $\frac{\text{Area } \Delta\text{XKL}}{\text{Area } \Delta\text{KLZ}} = \frac{\frac{1}{2}\text{XL} \times h_2}{\frac{1}{2}\text{LZ} \times h_2} = \frac{\text{XL}}{\text{LZ}}$ Area of $\Delta XKL = Area\ \Delta\ XKL$ common But Area $\Delta\ LKY = Area\ \Delta\ KLZ$ same base & height; LK// YZ $\frac{Area\ \Delta\ XKL}{Area\ \Delta LYK} = \frac{Area\ \Delta XKL}{Area\ \Delta KLZ}$ $\therefore \frac{XK}{KY} = \frac{XL}{LZ}$	✓ const ✓ $\frac{Area\ \Delta XKL}{Area\ \Delta LYK} = \frac{XK}{KY}$ ✓ $\frac{Area\ \Delta XKL}{Area\ \Delta KLZ} = \frac{XL}{LZ}$ ✓ R ✓ S (5)

10.2.1	$\frac{ED}{DC}=\frac{AT}{AC}=\frac{2}{3}$ line // to one side of △ $DE=\frac{2}{3}\times 9$ $=6$ $\therefore D$ is the midpoint of BE	✓ S ✓ R ✓DE = 6 (3)

10.2.2	BF= FT line from midpt drawn // to 2nd side bisct 3rd
	FD= $\frac{1}{2}$TE midpoint theorem
	$\therefore TE=4$
		✓ S/R
		✓ R
		✓ $TE=4$
		(3)

10.2.3(a)	$\frac{Area\ of\ \triangle ADC}{Area\ of\ \triangle ABD}$ $=\frac{DC}{BD}$ same height $=\frac{3y}{2y}$ $=\frac{3}{2}$	✓ same height ✓ $\frac{3}{2}$ (2)
10.2.3(b)	$\frac{\frac{1}{2}\times TC\times EC\times \sin C}{\frac{1}{2}\times AC\times BC\times \sin C}$ $=\frac{(x)(y)}{(3x)(5y)}$ $=\frac{1}{15}$	✓ $\frac{\frac{1}{2}\times TC\times EC\times \sin C}{\frac{1}{2}\times AC\times BC\times \sin C}$ ✓ $\frac{(x)(y)}{(3x)(5y)}$ ✓ $\frac{1}{15}$ (3)
		[16]

QUESTION 11

11.1	In $\triangle$TPS and $\triangle$ QSR $\frac{PS}{QS}=\frac{1,5}{4}=\frac{3}{8}$ $\frac{TP}{SR}=\frac{4,5}{12}=\frac{3}{8}$ $\frac{TS}{QR}=\frac{3,6}{9,6}=\frac{3}{8}$ $\therefore$ $\triangle$TPS$///\triangle$ QSR sides of triangles in proportion $\therefore P\hat{T}$S$=\hat{R}$ $\triangle$s are equiangular $\therefore TP$ is a tangent converse of tan chord theorem	✓ $\frac{PS}{QS}=\frac{1,5}{4}$ ✓ $\frac{TP}{SR}=\frac{4,5}{12}$ ✓ $\frac{TS}{QR}=\frac{3,6}{9,6}$ ✓✓ S/R ✓ $P\hat{T}$S$=\hat{R}$ ✓R (7)
11.2	$\hat{P}=Q\hat{S}R$ $\triangle$s are equiangular QS|| TP corres. <s are = $\frac{TQ}{9,6}=\frac{1,5}{12}$ proportion theorem; PT||SQ $\therefore TQ\ =1,2$	✓ S/R ✓ S/R ✓✓ S/R ✓ $TQ\ =1,2$ (5)
		[12]

TOTAL: 150