QUESTION 2

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1 Calculate:

2.1.1 The mean of the data (2)

2.1.2 The interquartile range of the data (3)

2.2 The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours? (2)

2.3 Draw a box and whisker diagram to represent the data on the number line provided in the ANSWER BOOK. (3)

2.4 The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15 ; 21 ; 23,5 ; 26 ; 38).

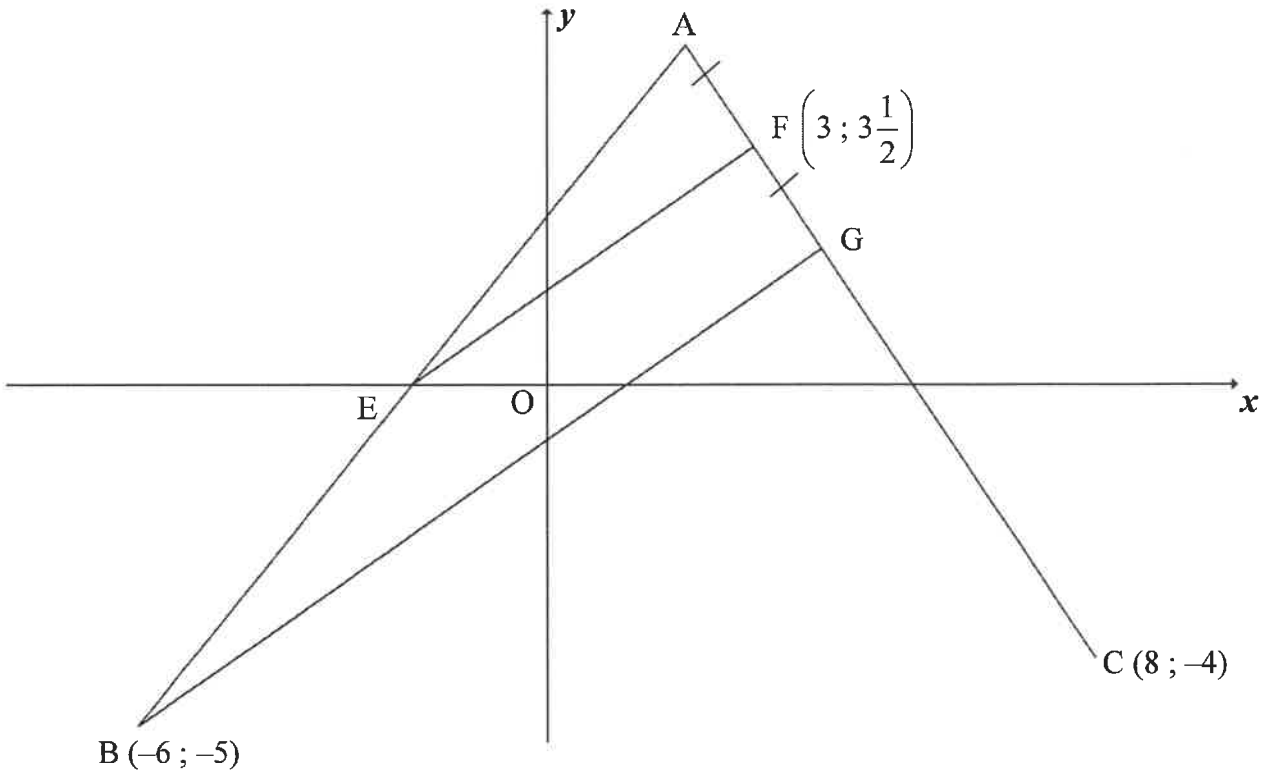
2.4.1 Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles? (1)

2.4.2 The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prizewinners? Motivate your answer. (2)

[13]

QUESTION 3

In the diagram, A, B(–6 ; –5) and C(8 ; –4) are points in the Cartesian plane. $F\left(3;3\frac{1}{2}\right)$ and G are points on line AC such that AF = FG. E is the *x*-intercept of AB.



3.1 Calculate:

3.1.1 The equation of AC in the form $y = mx + c$ (4)

3.1.2 The coordinates of G if the equation of BG is $7x - 10y = 8$ (3)

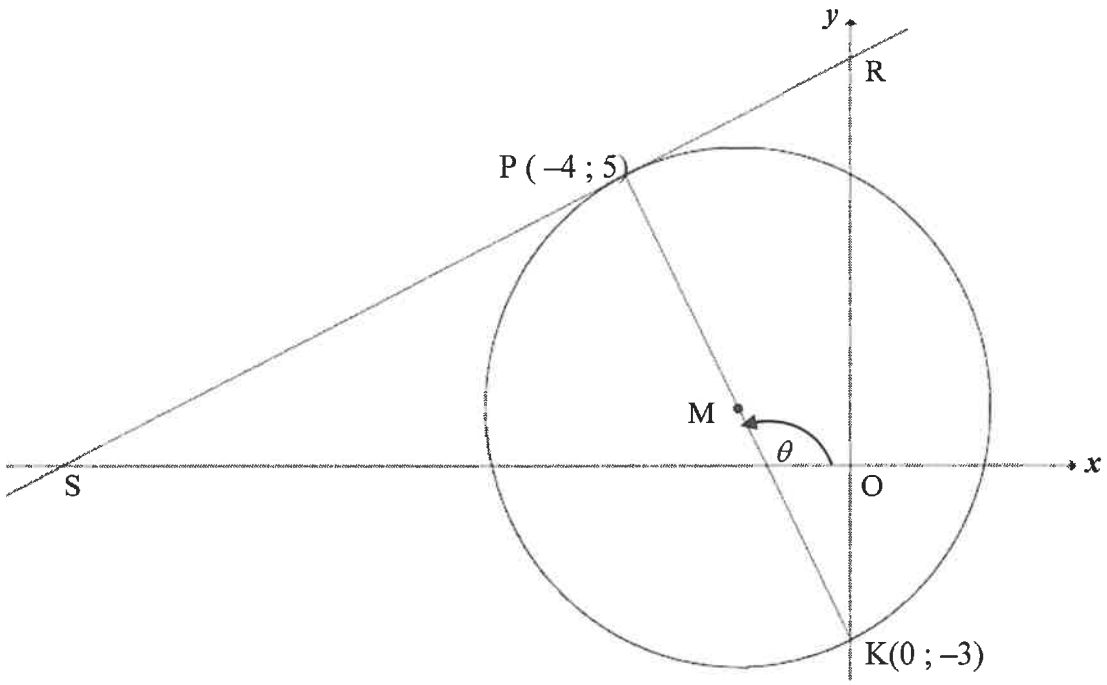
3.2 Show by calculation that the coordinates of A is (2 ; 5). (2)

3.3 Prove that EF || BG. (4)

3.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D. (4)
[17]

QUESTION 4

In the diagram, P(–4 ; 5) and K(0 ; –3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



4.1 Determine:

4.1.1 The gradient of SR (4)

4.1.2 The equation of SR in the form $y = mx + c$ (2)

4.1.3 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

4.1.4 The size of $P\hat{K}R$ (3)

4.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$ (2)

4.2 Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

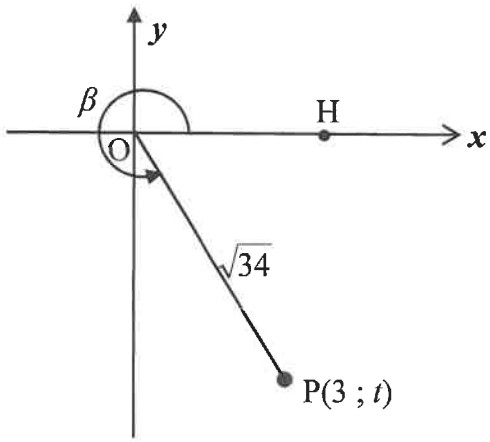
4.3 Calculate the area of ΔSMK. (5)
[23]

QUESTION 5

5.1 Given: $\dfrac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio. (6)

5.2 In the diagram, P(3 ; t) is a point in the Cartesian plane. OP $=\sqrt{34}$ and $\hat{HOP}=\beta$ is a reflex angle.



Without using a calculator, determine the value of:

5.2.1 t (2)

5.2.2 $\tan\beta$ (1)

5.2.3 $\cos 2\beta$ (4)

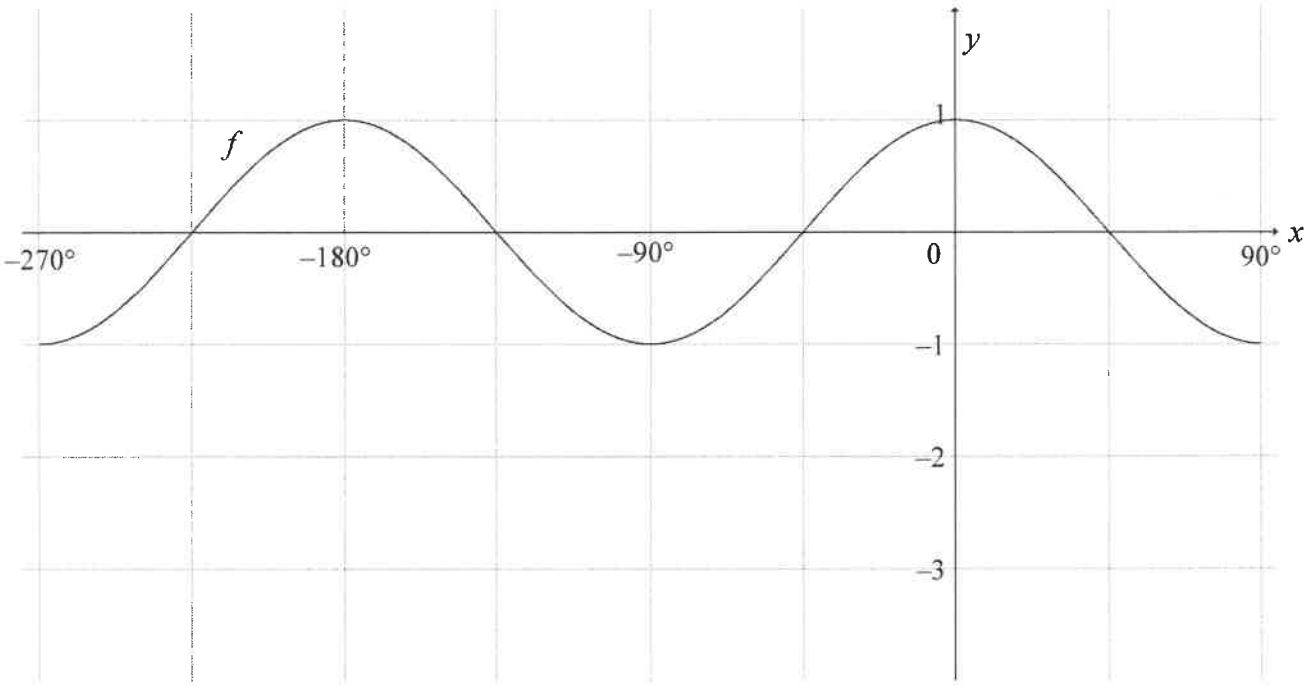
5.3 Prove:

5.3.1 $\sin(A+B)-\sin(A-B)=2\cos A.\sin B$ (2)

5.3.2 **Without using a calculator,** that $\sin 77°-\sin 43°=\sin 17°$ (4)
[19]

QUESTION 6

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270°; 90°]$.



6.1 Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270°; 90°]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)

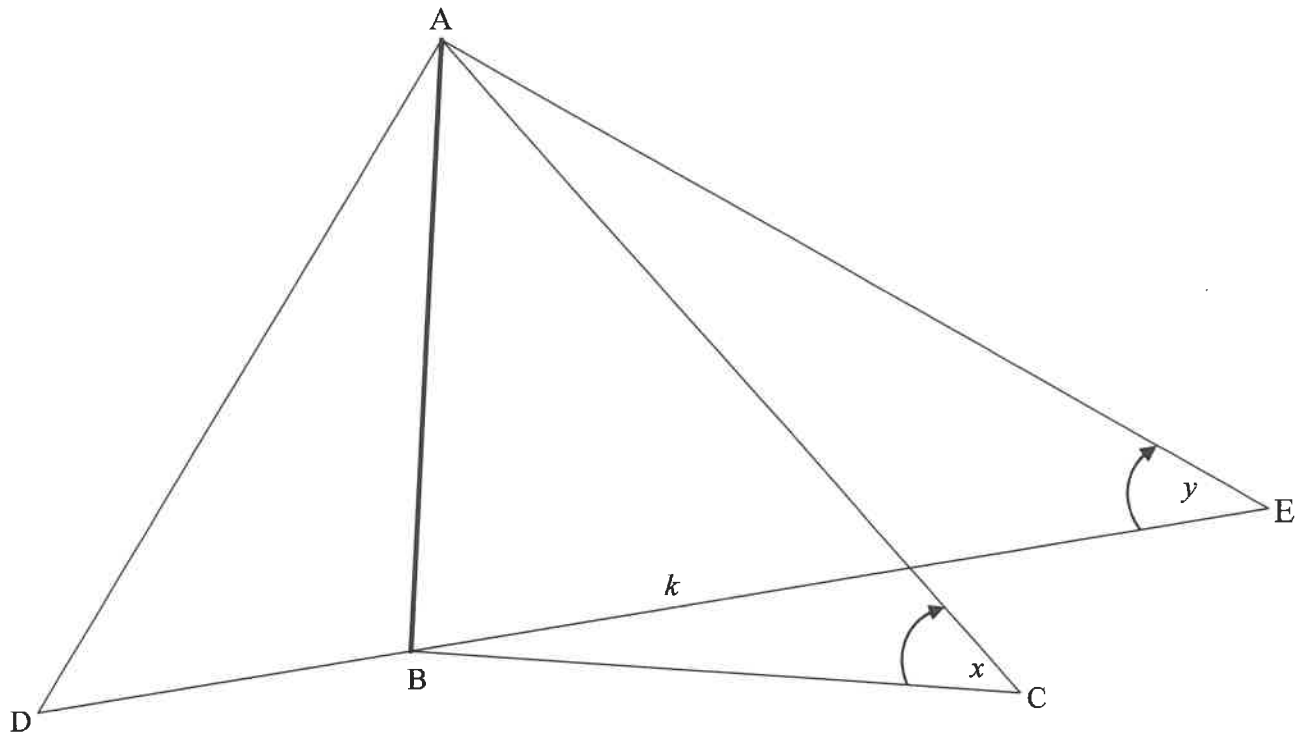
6.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x = \dfrac{-1+\sqrt{5}}{2}$. (4)

6.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x \in [-270°; 90°]$. (4)

[12]

QUESTION 7

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



7.1 Write down the size of $A\hat{B}C$. (1)

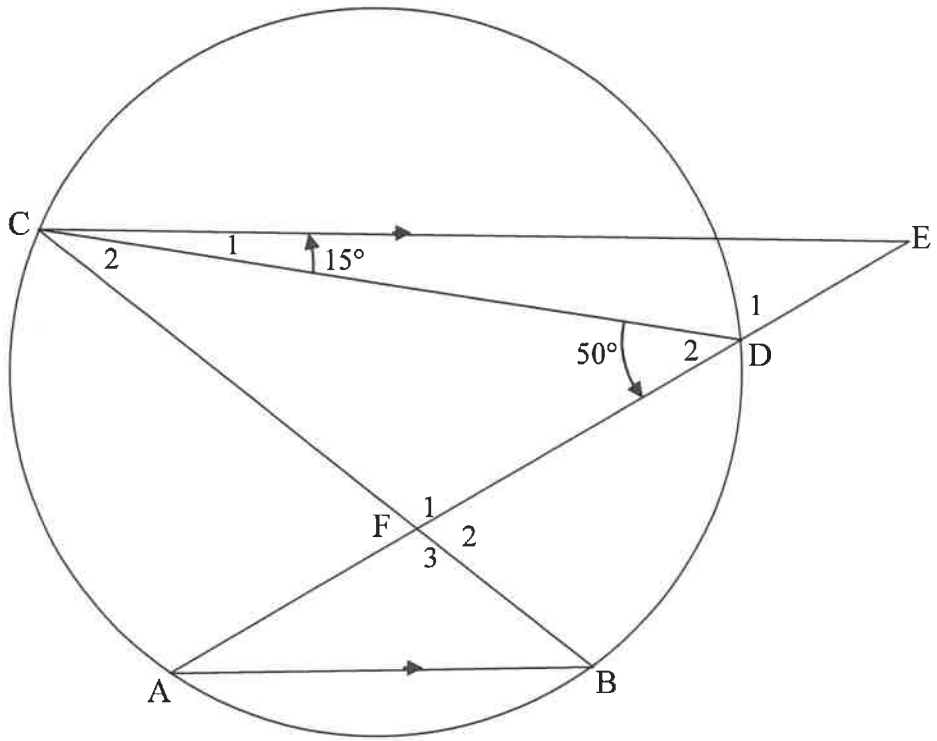
7.2 Show that $AC = \dfrac{k.\tan y}{\sin x}$ (4)

7.3 If it is further given that $D\hat{A}C = 2x$ and AD = AC, show that the distance DC between the players at D and C is $2k\tan y$. (5)
[10]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram, points A, B, D and C lie on a circle. CE || AB with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



8.1 Calculate, with reasons, the size of:

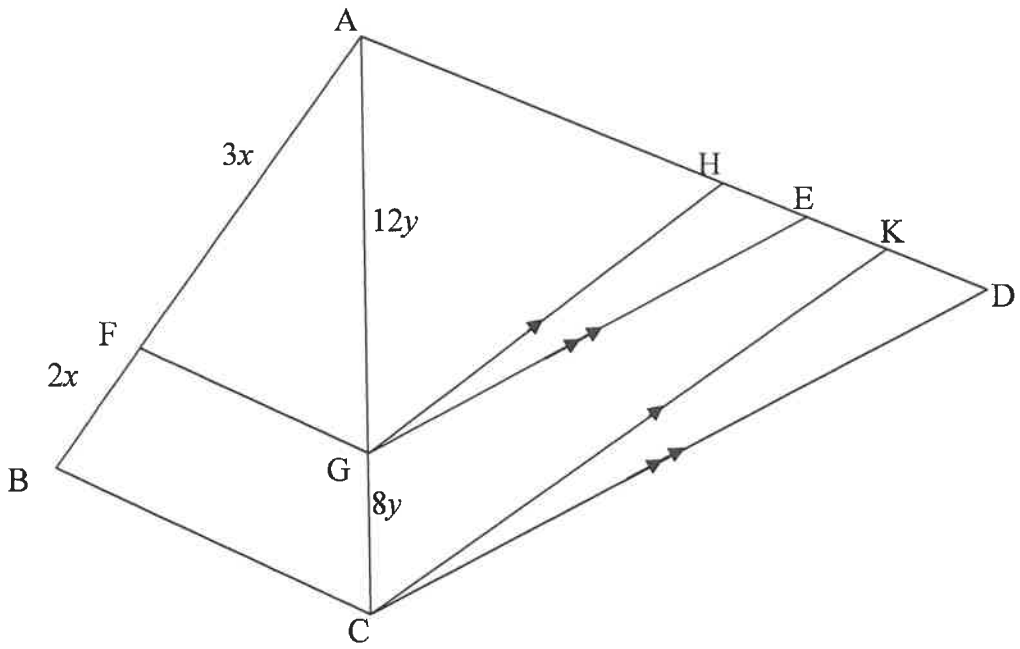
8.1.1 $\hat{A}$ (3)

8.1.2 $\hat{C}_2$ (2)

8.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2)
[7]

QUESTION 9

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



9.1 Prove that:

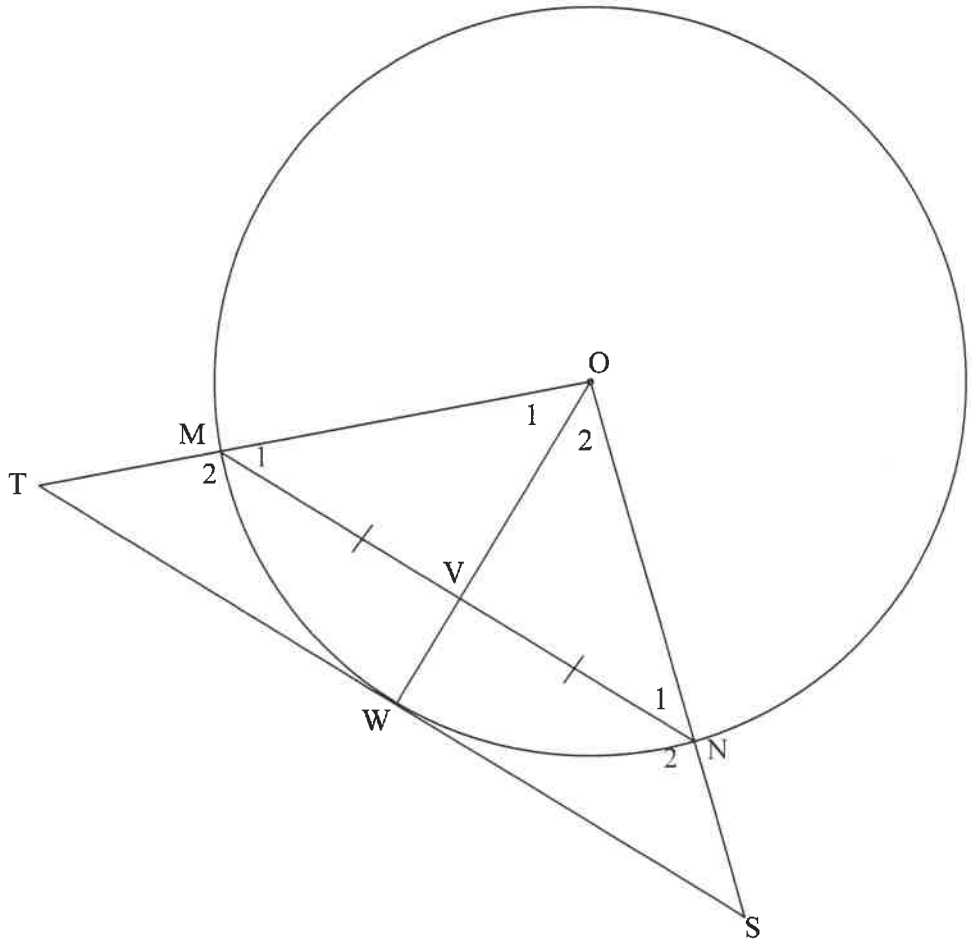
9.1.1 FG || BC (2)

9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

9.2 If it is further given that AH = 15 and ED = 12, calculate the length of EK. (5)
[10]

QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



10.1 Give a reason why OV ⊥ MN. (1)

10.2 Prove that:

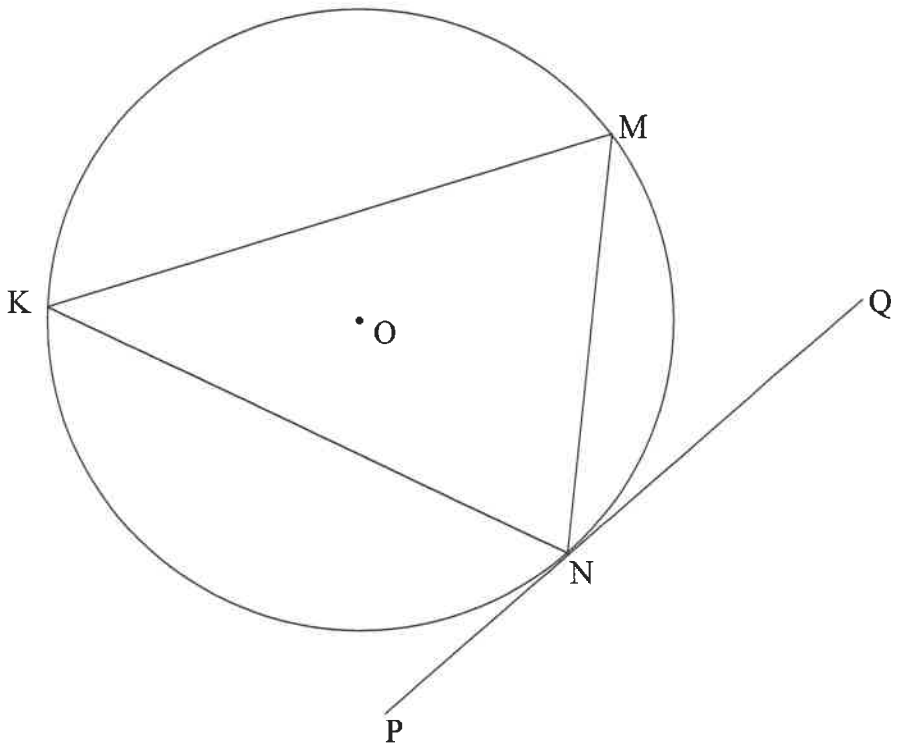
10.2.1 MN || TS (2)

10.2.2 TMNS is a cyclic quadrilateral (4)

10.2.3 OS . MN = 2ON . WS (5)
[12]

QUESTION 11

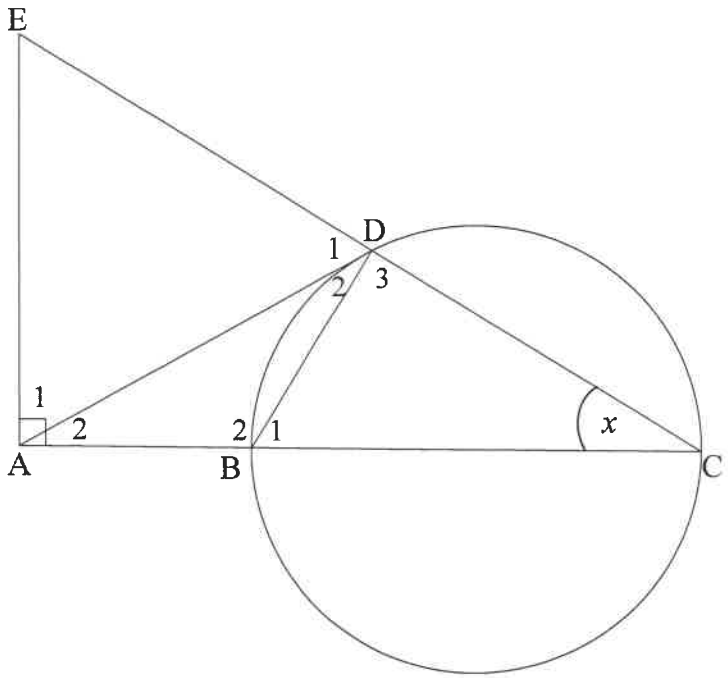
11.1 In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.



Prove the theorem which states that $M\hat{N}Q = \hat{K}$. (5)

11.2 In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA ⊥ AC. BD is drawn.

Let $\hat{C} = x$.



11.2.1 Give a reason why:

(a) $\hat{D}_3 = 90°$ (1)

(b) ABDE is a cyclic quadrilateral (1)

(c) $\hat{D}_2 = x$ (1)

11.2.2 Prove that:

(a) AD = AE (3)

(b) $\Delta ADB \,|||\, \Delta ACD$ (3)

11.2.3 It is further given that BC = 2AB = 2r.

(a) Prove that $AD^2 = 3r^2$ (2)

(b) Hence, prove that ΔADE is equilateral. (4)

[20]

TOTAL: 150

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INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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