GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

PROOFREAD ☐	DATE ___________
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PROOFREAD ☐	DATE ___________
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MATHEMATICS
(Second Paper)

TIME: 3 hours

MARKS: 150

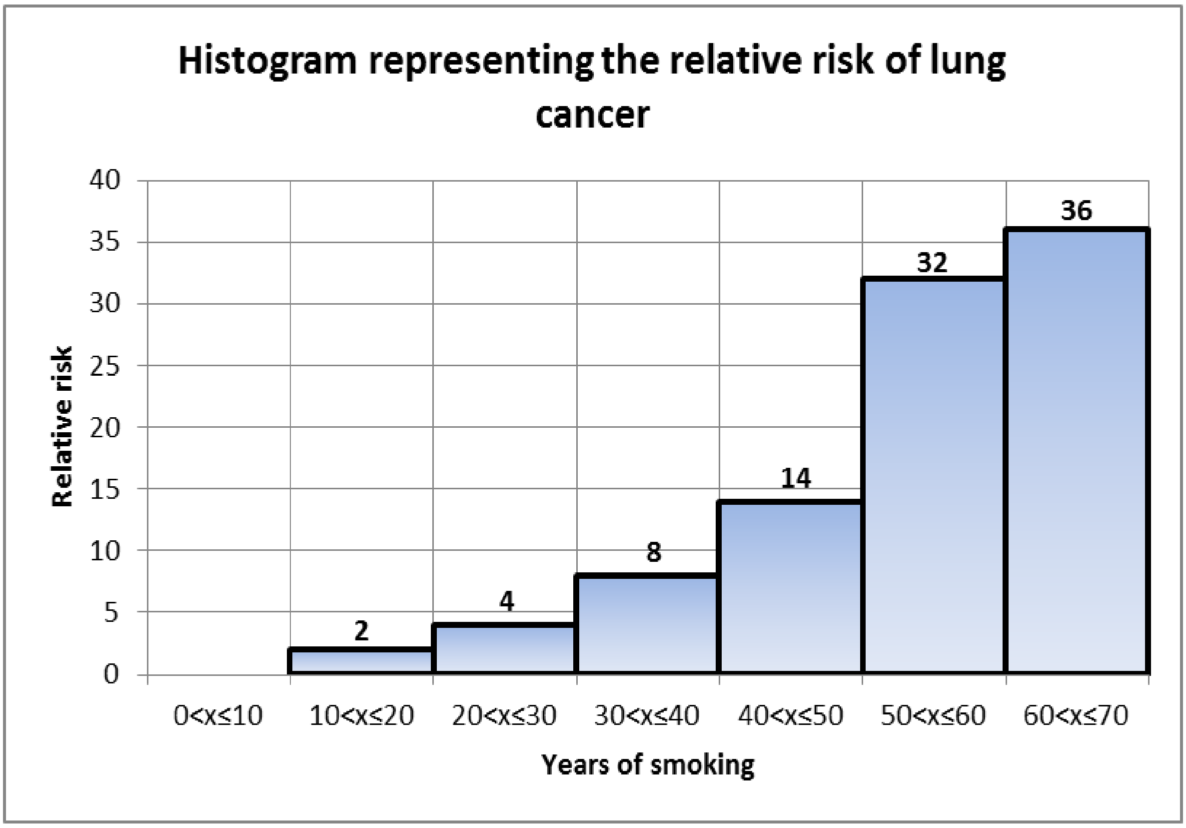
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Six DIAGRAM SHEETS for QUESTIONS 1.2, 2.1, 2.4, 7.1, 9.2, 10, 11.1, 11.2, 12.2 and 12.3 and the INFORMATION SHEET are attached at the end of this question paper.
9. Write your name in the space provided on the DIAGRAM SHEETS and hand them in with your ANSWER BOOK.
10. Number the answers correctly according to the numbering system used in this question paper.
11. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

Globally, cigarette smoking is killing people by the millions every single year.
The histogram below represents the total number of years as a smoker (25 or more cigarettes per day) against the relative risk to contract lung cancer.
The risk of contracting lung cancer increases with both duration and intensity of smoking.



1.1 State whether the distribution of the data is normal, positively skewed or negatively skewed . (1)

1.2 On **DIAGRAM SHEET 1**, draw the ogive (cumulative frequency polygon) of the above data. (4)

1.3 Indicate clearly on the sketch drawn on **DIAGRAM SHEET 1,** where the median can be read off. (1)

[6]

QUESTION 2

During the month of June patients visited a number of medical facilities for treatment. The table shows the number of patients treated on certain dates during the month of June.

Dates in the month of June	3	5	8	12	15	19	22	26
Number of patients treated	270	275	376	420	602	684	800	820

2.1 On **DIAGRAM SHEET 2**, draw a scatter plot of the given data. (3)

2.2 Determine the equation of the least squares regression line of patients treated (y) against date (x). (3)

2.3 Estimate how many patients have been treated on the 24th of June. (2)

2.4 Draw the least squares regression line on the grid on DIAGRAM SHEET 2. (3)

2.5 Calculate the correlation coefficient of the data. Comment on the strength of the relationship between the variables. (3)

2.6 The statistics gathered for Youth Day, the 16th of June was (16 ; 102). Comment on this point and justify your answer. (2)

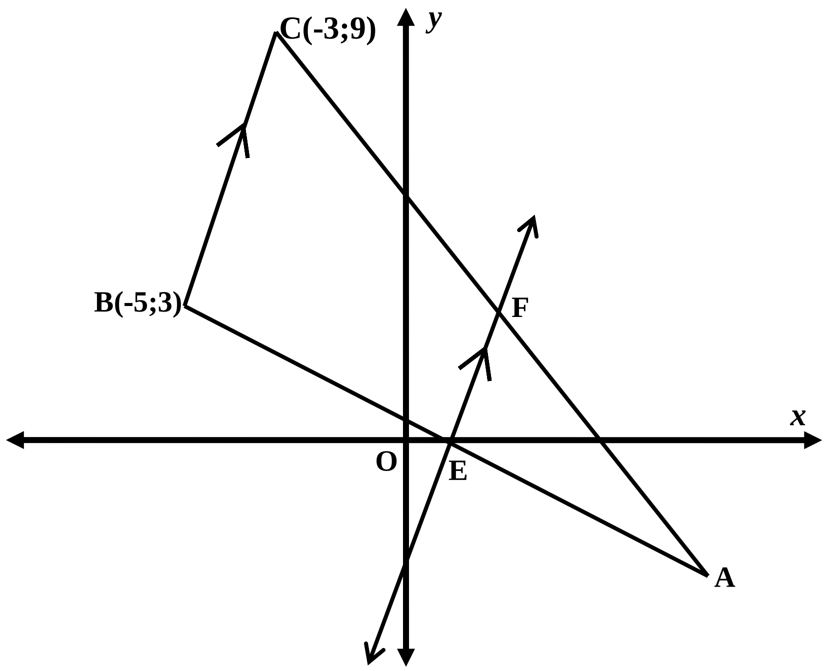
[16]

QUESTION 3

3.1 The points Q (1 ; 3) and R (7 ; 0) lie on the line l_1. The length of QR is $a\sqrt{5}$.
Determine the value of a. (3)

3.2 Three straight lines AB, RS and $x = -3$ intersect each other.
The equation of AB is $3x + ty = -2$ with $t \neq 0$, and the equation of RS is
$y = -\frac{2}{3}x + 2$.
Calculate t. (4)

3.3 In the diagram below, EF || BC and the vertices of ΔABC are A, B (– 5 ; 3) and C (– 3 ; 9). The x-intercept at point E of line AB is (1 ; 0).



3.3.1 Calculate θ, the angle of inclination of line EB. (3)

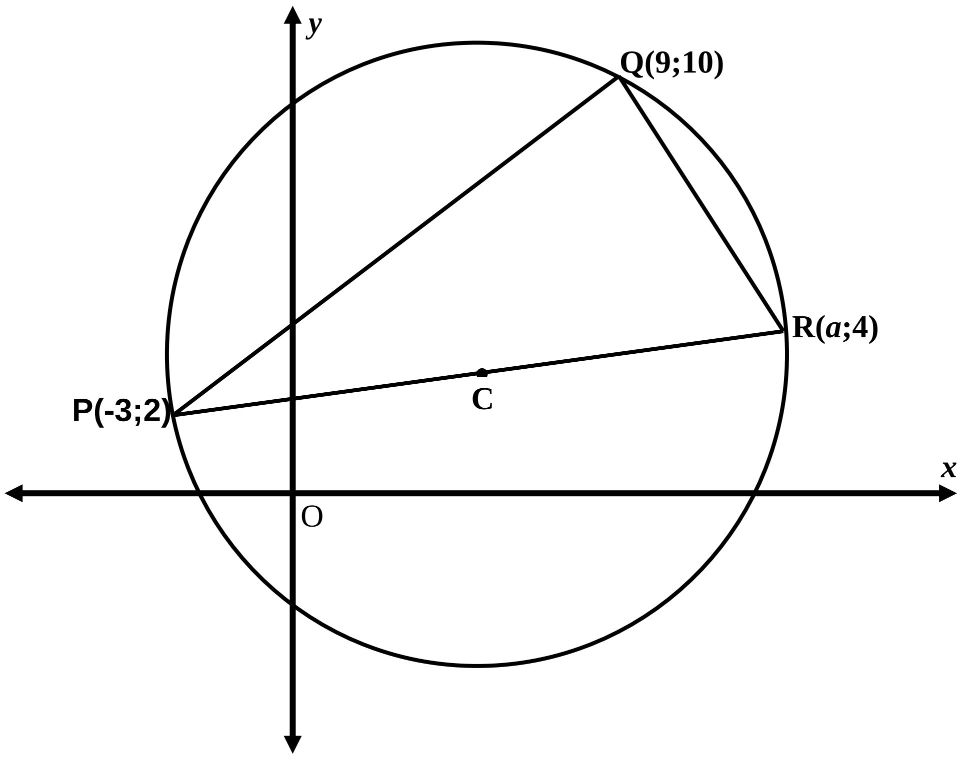
3.3.2 Determine the equation of line EF, which is parallel to BC and passes through point E. (4)

3.3.3 Calculate the size of $E\hat{B}C$. (3)

[17]

QUESTION 4

4.1 The points P(−3 ; 2), Q(9 ; 10) and R(a ; 4) lie on the circumference of the circle, as shown in the figure below. PR is a diameter of the circle with centre C.



4.1.1 Show that a = 13. (3)

4.1.2 Determine the equation of the circle in the form:
$$(x-a)^2+(y-b)^2=r^2$$
(4)

4.2 Two circles with centres A and B are given below.

Circle A: $(x-2)^2+(y-3)^2=9$

Circle B: $(x-1)^2+(y+1)^2=16$

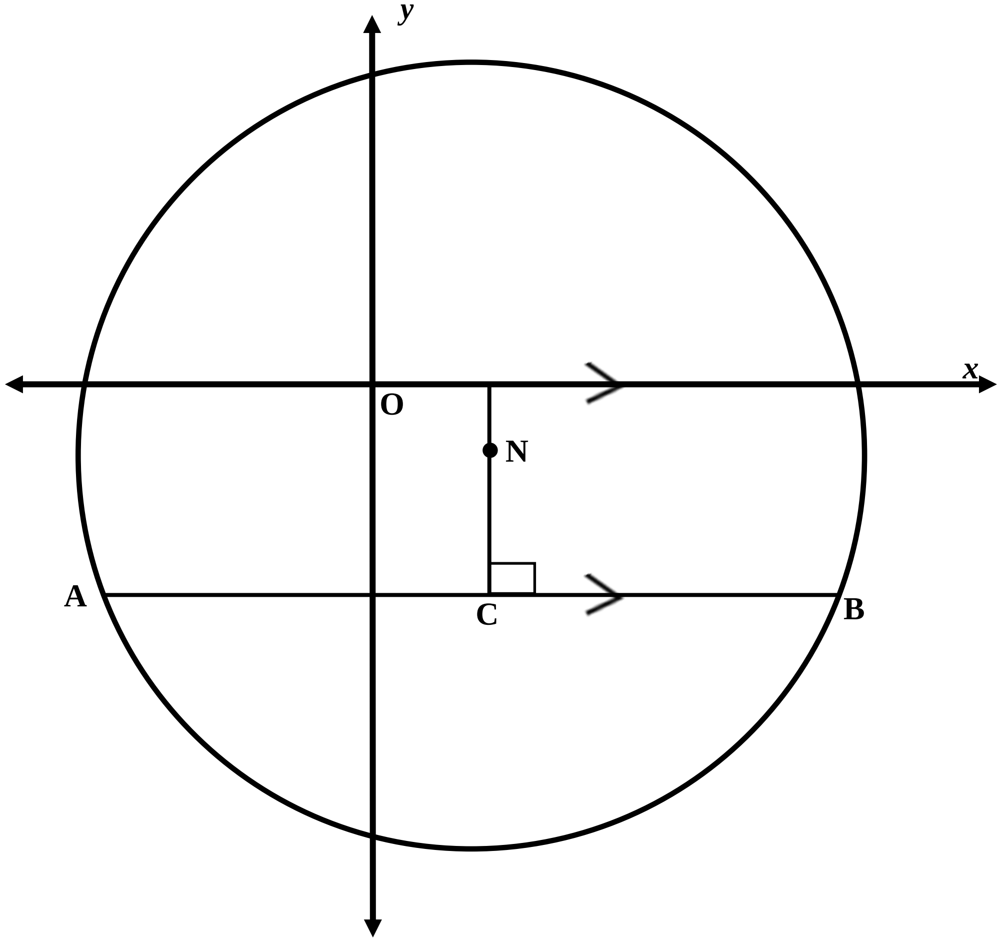
Without solving for x and y, determine if the circles

- intersect each other at 2 points.
- touch each other.
- do not intersect each other.

Show all your calculations. (6)

4.3 The figure below shows a sketch of the circle with centre N and equation $x^2 - 4x + y^2 + 2y = \frac{149}{4}$.

Chord AB of circle N is parallel to the x-axis and lies below the x-axis. The length of AB is 12 units.



4.3.1 Determine the coordinates of N. (3)

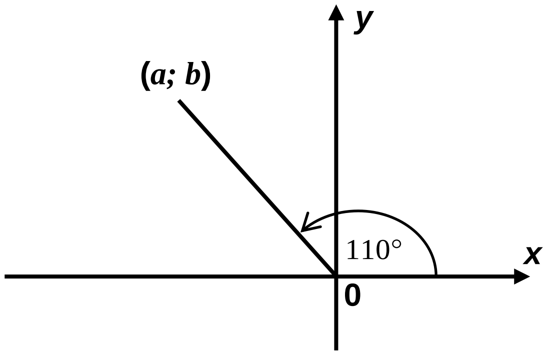
4.3.2 Determine the radius of the circle. (1)

4.3.3 Calculate the coordinates of A and B. (5)

[22]

QUESTION 5

5.1 In the Cartesian plane below, the point (a ; b) and the angle of $110°$ are shown.



5.1.1 Complete the following: $\dfrac{b}{a} = ...110°$. (1)

5.1.2 Determine, rounded off to two decimal places the value of:

$$\frac{b}{\sqrt{a^2 + b^2}}$$ (2)

5.2 Given: $\sin 56° = q$

Determine **without using a calculator**, the value of the following in terms of q:

5.2.1 $\cos 146°$ (2)

5.2.2 $\sin 112°$ (3)

[8]

QUESTION 6

6.1 Express $\cos(P+Q)$ in terms of trigonometric ratios of P and Q. (1)

6.2 Hence, derive a formula, which expresses $\cos 2P$ in terms of $\cos P$. (3)

6.3 Given: $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1}$

6.3.1 Prove that: $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = \tan x$ (5)

6.3.2 Hence, evaluate $\tan 22{,}5°$. (Leave your answer in SURD form). (3)

6.4 Given the expression: $\sin 2x.\cos 2x$

6.4.1 Calculate the maximum value of the above expression. (3)

6.4.2 Calculate the first negative value of x for which the expression has this maximum value. (4)

[19]

QUESTION 7

7.1 Use the system of axes on **DIAGRAM SHEET 3** to sketch the graphs of:

$$f(x) = -\frac{1}{2}\sin(x + 30°) \text{ and } g(x) = \cos 2x \text{ if } -180° \le x \le 180°$$ (6)

7.2 Write down the period of g. (1)

7.3 Graph h is obtained when the y-axis for f is moved $120°$ to the left.
Give the equation of h in the form $h(x) = ...$ (2)

7.4 Determine the general solution of:

$$\cos 2x = 1$$ (2)

[11]

QUESTION 8

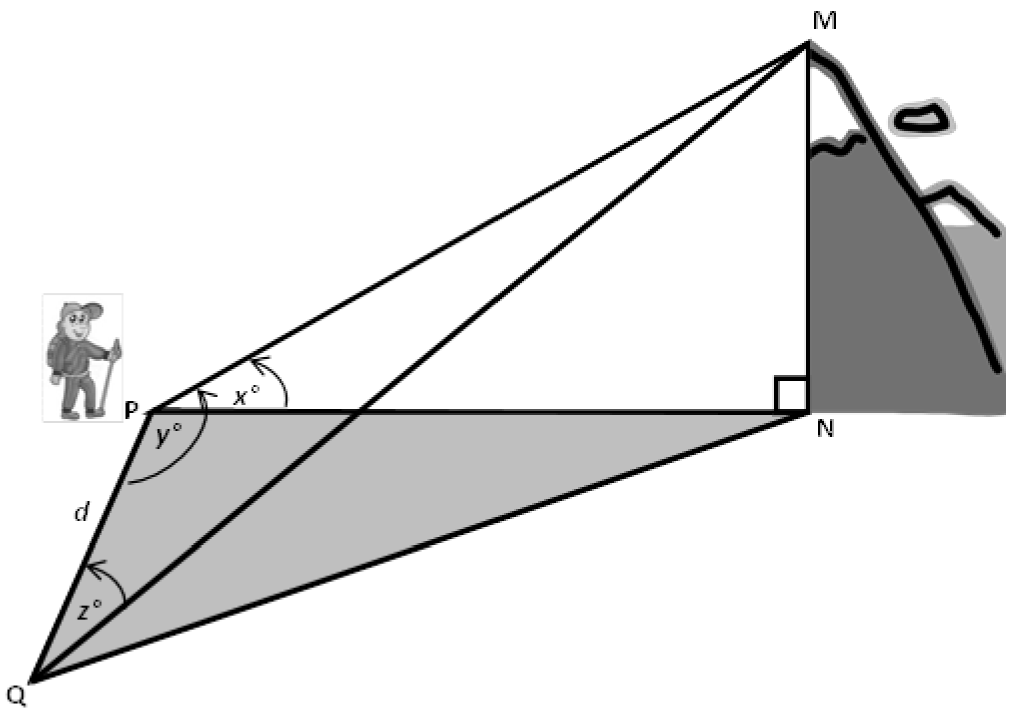
A mountain climber wants to determine the height, MN, of a mountain.

He uses a theodolite (instrument used to measure angles). The mountain climber is standing on flat ground. P is in the same horizontal plane as the foot of the mountain N.

He measures the following angles from P:

- The angle of elevation of the top of the mountain M, is $x°$
- $M\hat{P}Q$ is $y°$

He then walks d metres to point Q, which is also in the same horizontal plane as P and N, and measures $P\hat{Q}M$ which is $z°$.



8.1 Show that: $MN = \dfrac{d \sin x \sin z}{\sin(y+z)}$ (4)

8.2 Calculate MN, **without using a calculator** if $d = 1000$ m, $x = 90° - y$ and $y = z$. (3)

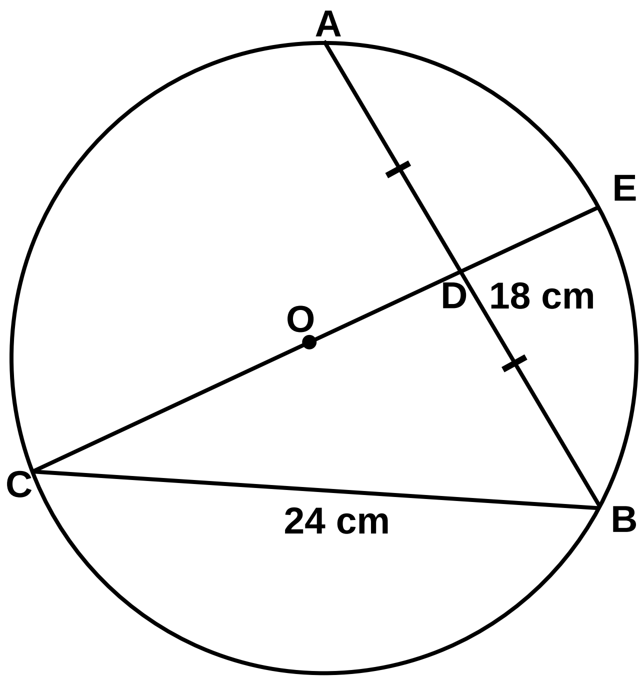
[7]

QUESTION 9

9.1 Complete the following so that the Euclidian Geometry statement is true: (1)

A line drawn from the centre of a circle to the midpoint of the chord is ....to the chord

9.2 In the circle with centre O, chord AB = 18cm and AD = DB.
Chord CB = 24 cm.



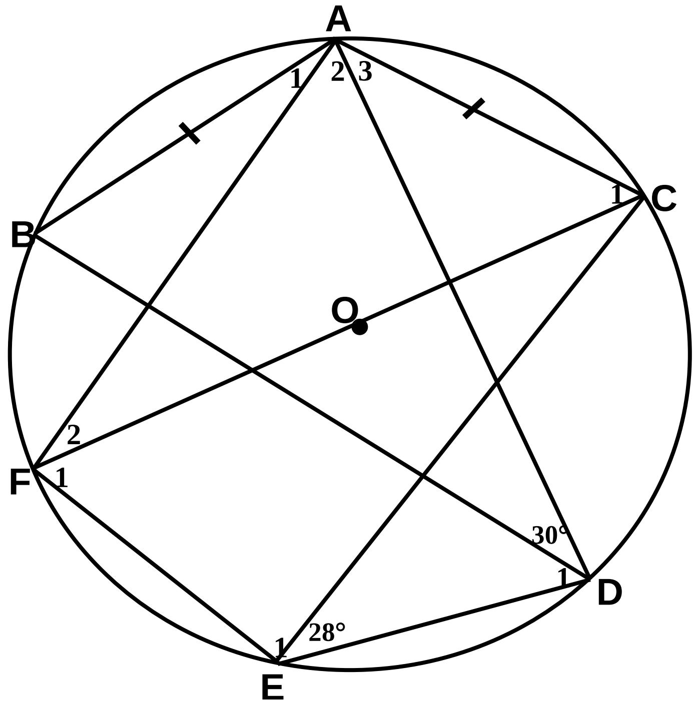
9.2.1 Calculate the length of CD. Leave the answer in simplest surd form. (4)

9.2.2 If $\frac{CD}{DE} = 3$, calculate the length of DE. (2)

[7]

QUESTION 10

In the diagram, O is the centre of the circle. Chords AB = AC. $C\hat{E}D = 28°$ and $A\hat{D}B = 30°$



Calculate, with reasons, the sizes of the following angles:

10.1 $\hat{E}_1$ (2)

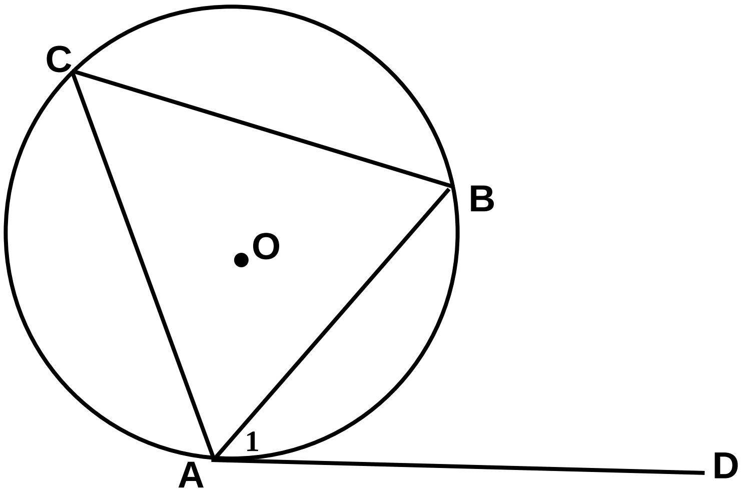
10.2 $\hat{A}_2$ (3)

10.3 $\hat{F}_2$ (2)

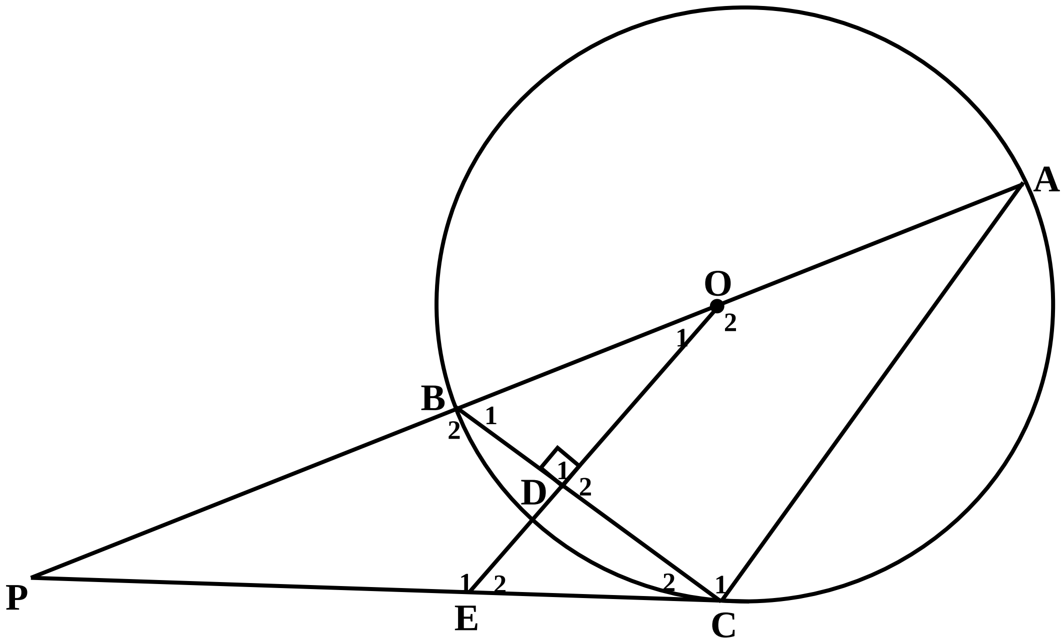
[7]

QUESTION 11

11.1 Use the diagram on **DIAGRAM SHEET 5**, to prove the theorem that states that $\hat{A}_1 = \hat{C}$ (5)



11.2 In the diagram, AB is a diameter of circle, centre O. AB is produced to P. PC is a tangent to the circle at C. $OE \perp BC$ at D.



11.2.1 Prove, with reasons, that EO || CA. (4)

11.2.2 If $\hat{C}_2 = x$, name with reasons, two other angles each equal to x. (3)

11.2.3 Calculate the size of $\hat{P}$ in terms of x. (2)

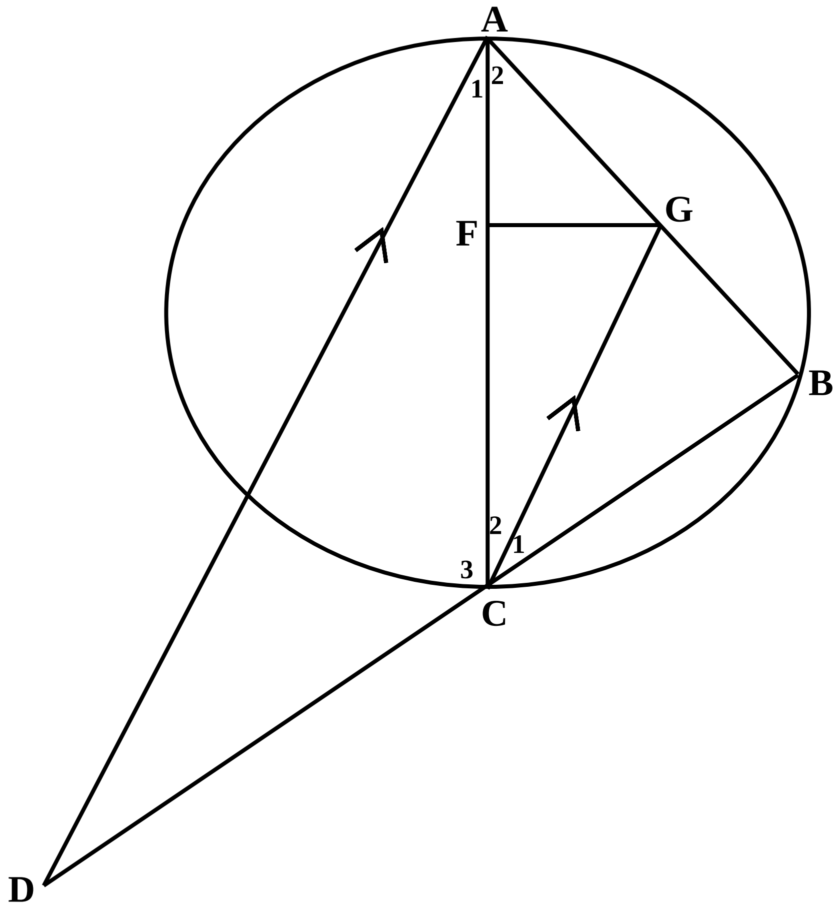
[14]

QUESTION 12

12.1 Complete the following so that the Euclidian Geometry statement is true:

A line drawn parallel to one side of a triangle divides the other two sides ……. (1)

12.2 In the diagram below, CG bisects $A\hat{C}B$. AD || GC.

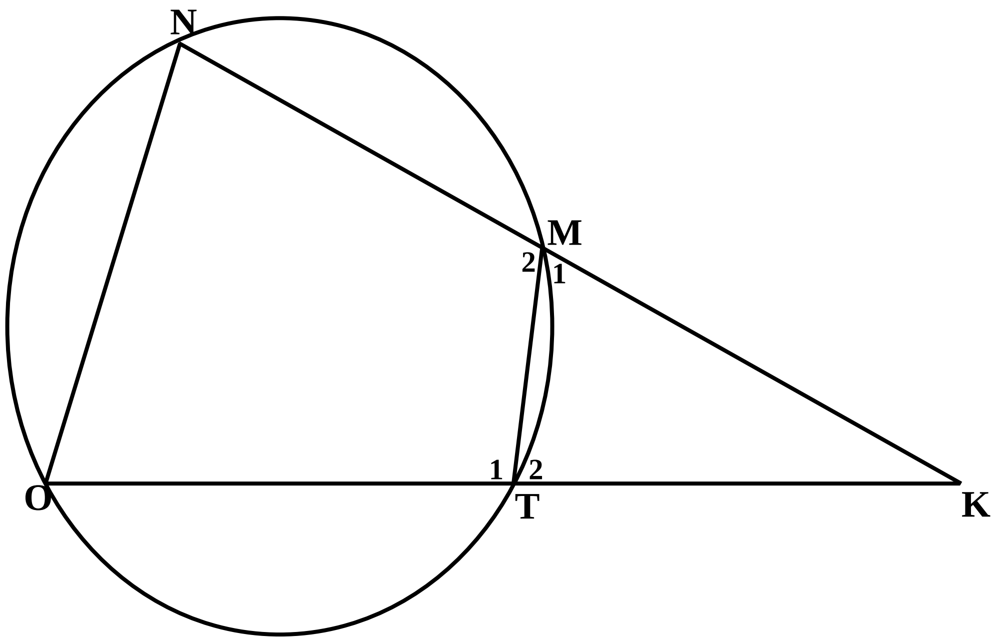


Prove, with reasons, that:

12.2.1 AC = DC (4)

12.2.2 $\frac{BC}{AC} = \frac{BG}{AG}$ (3)

12.3 In the diagram, KMN and KTO are two secants of a circle.



12.3.1 Prove that $\Delta MTK \;|||\; \Delta ONK$. (3)

12.3.2 Hence, prove that KM.KN = KT.KO (1)

12.3.3 Calculate KT if OT = 6 units, MN = 3 units and MK = 5 units. (4)

[16]

TOTAL: 150

NAME:

DIAGRAM SHEET 1

QUESTION 1.2

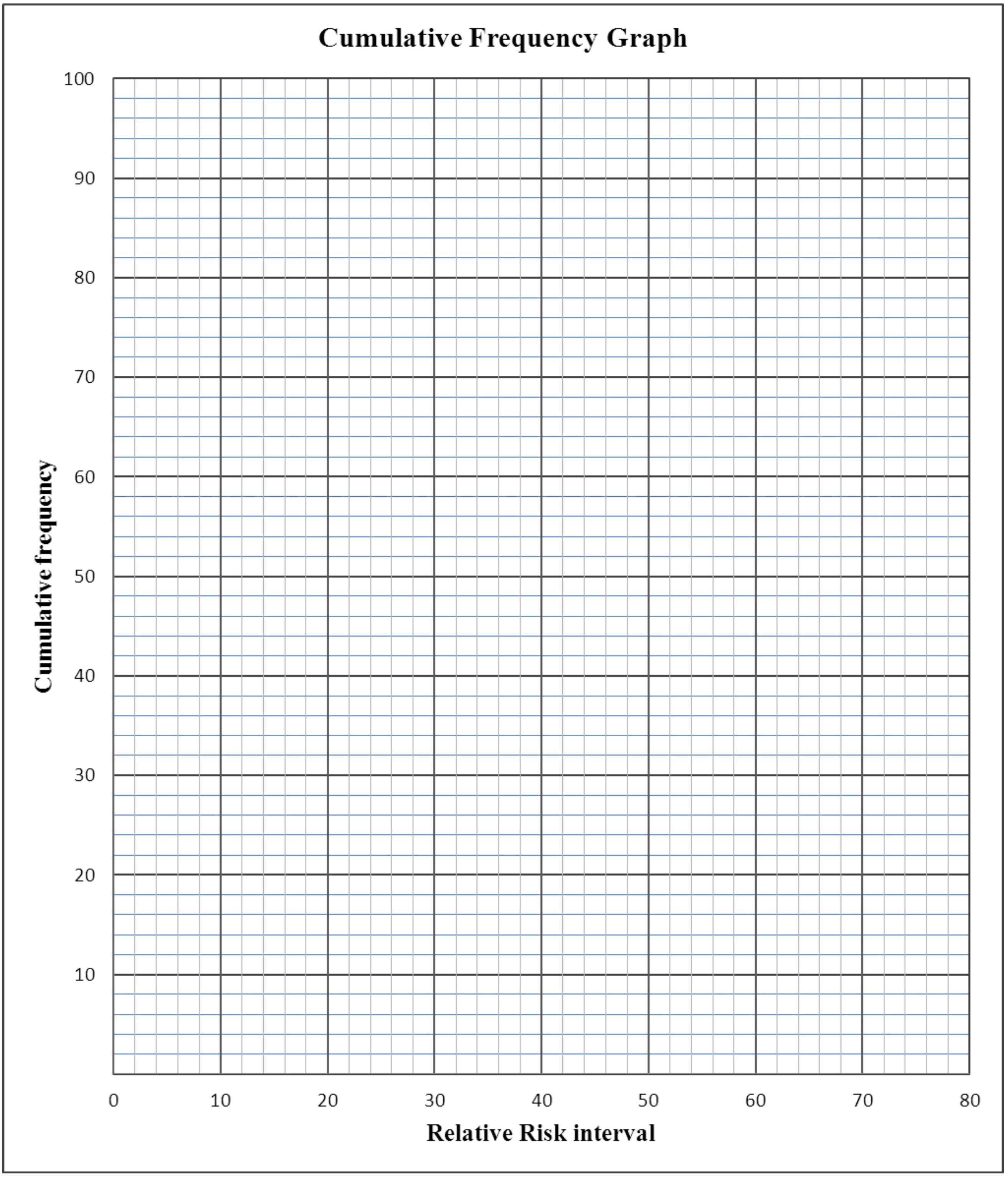


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DIAGRAM SHEET 2

QUESTIONS 2.1 AND 2.4

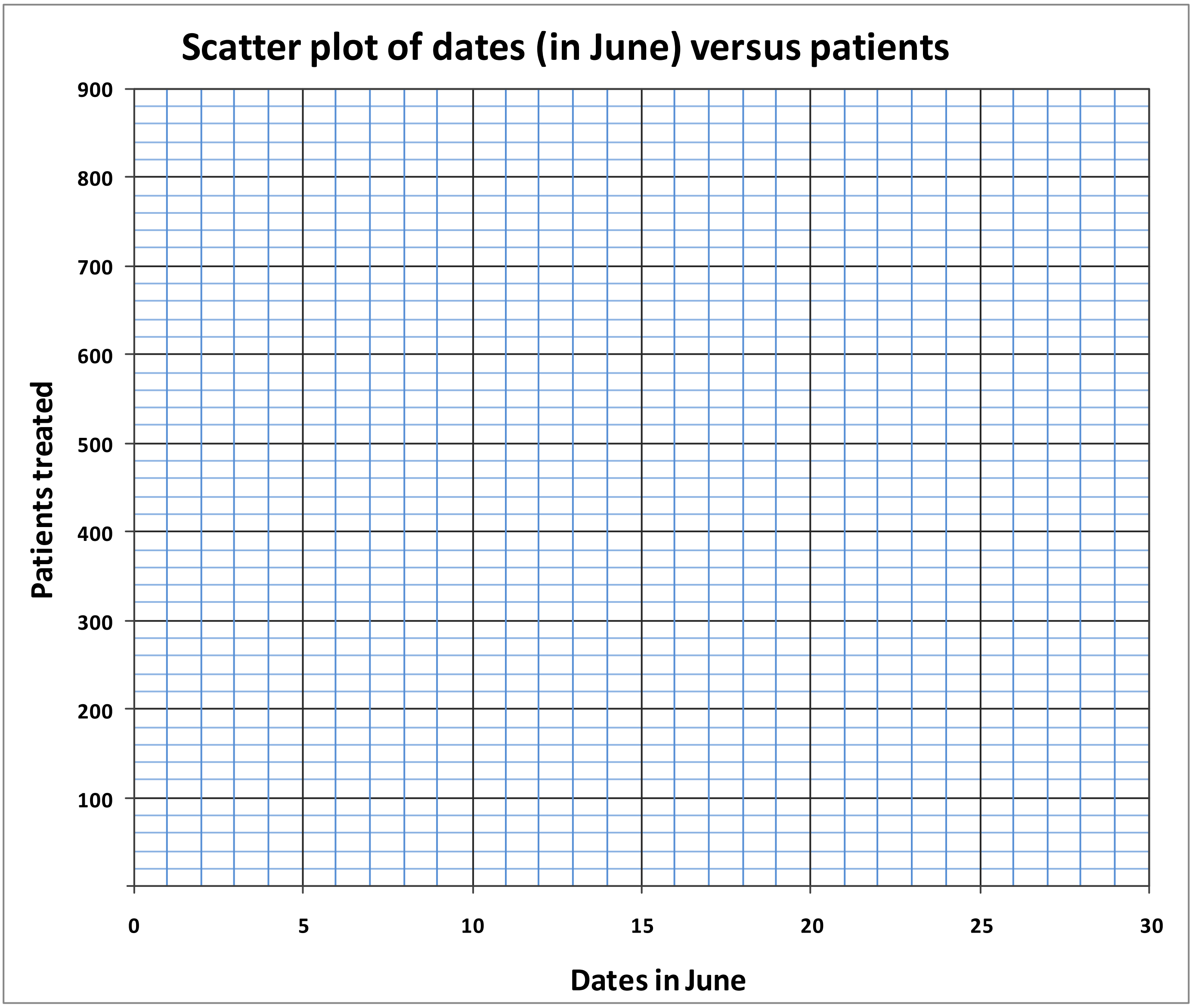

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DIAGRAM SHEET 3

QUESTION 7.1

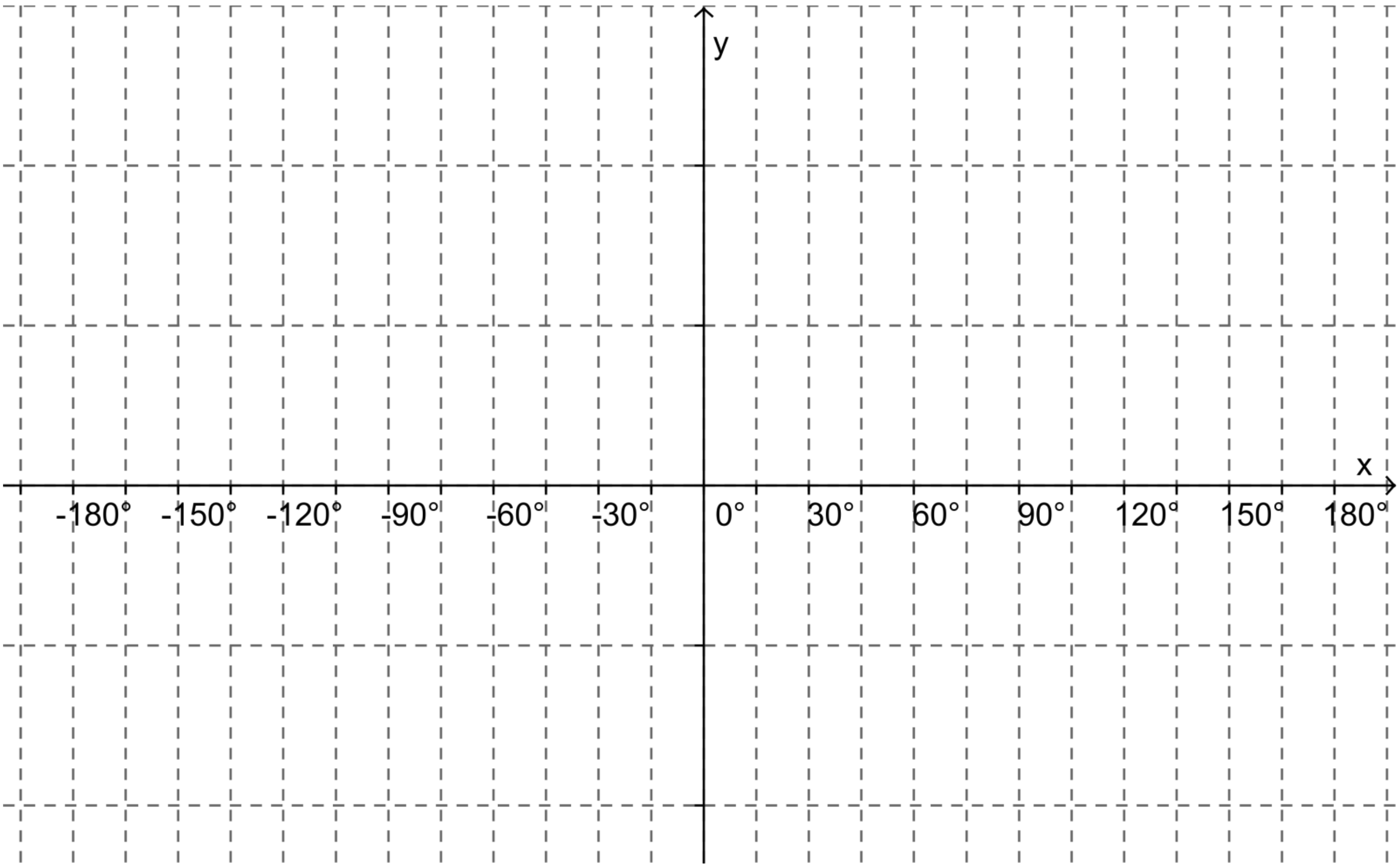

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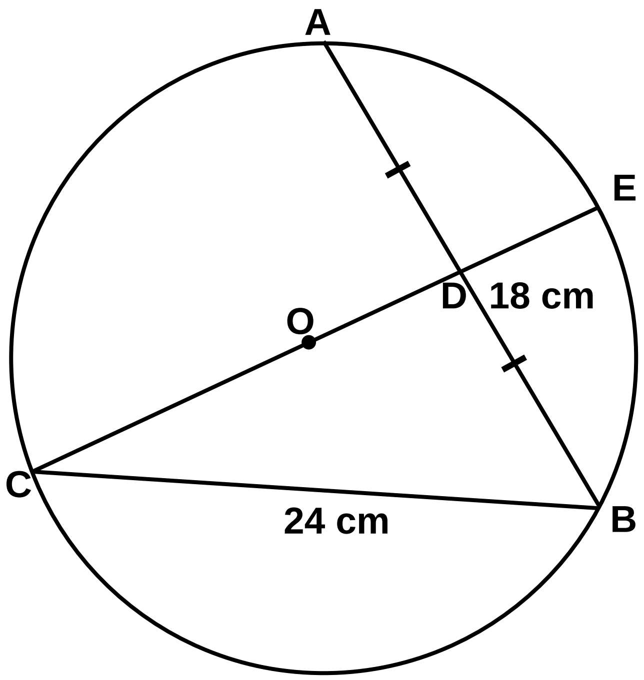
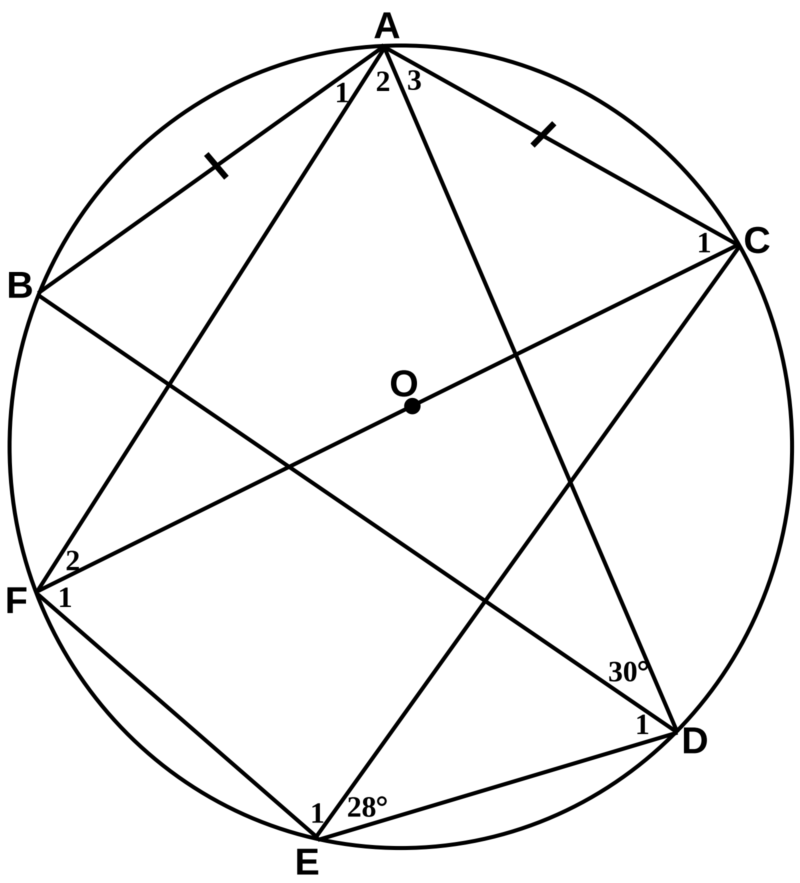
DIAGRAM SHEET 4

QUESTION 9.2



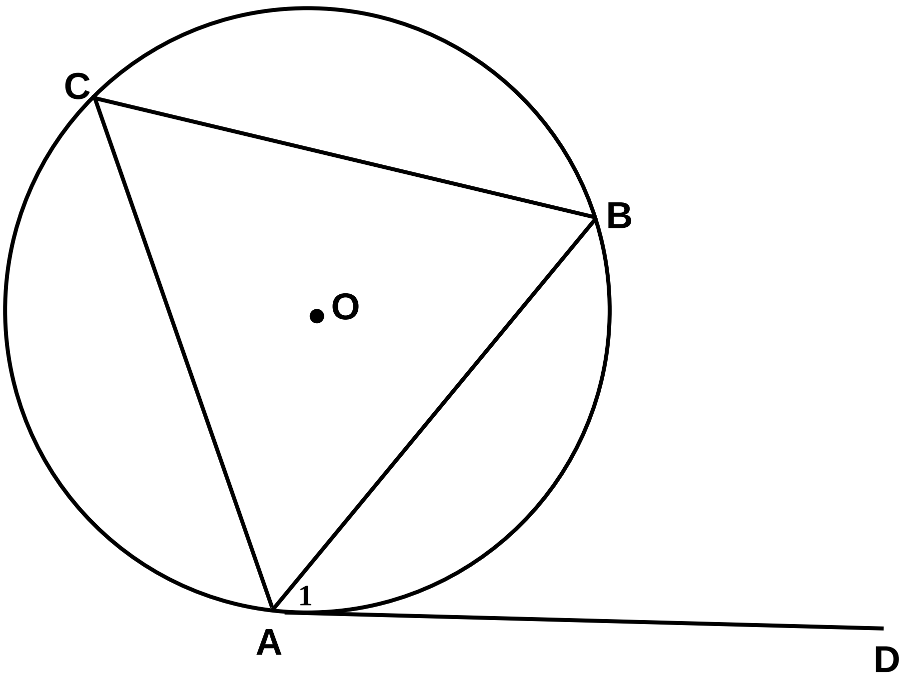
QUESTION 10



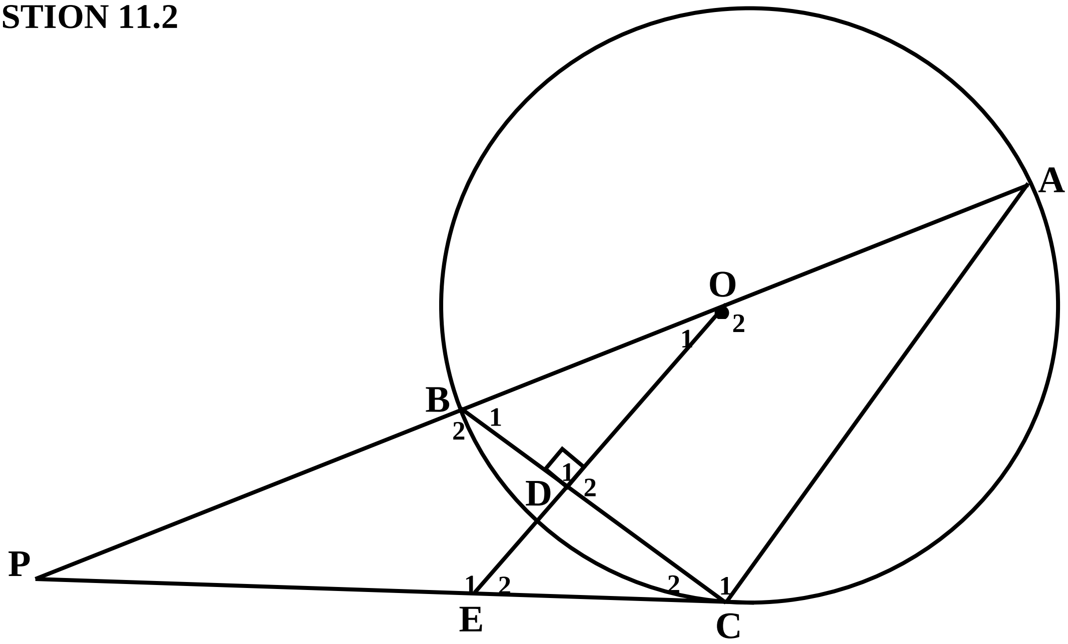
NAME:

DIAGRAM SHEET 5

QUESTION 11.1



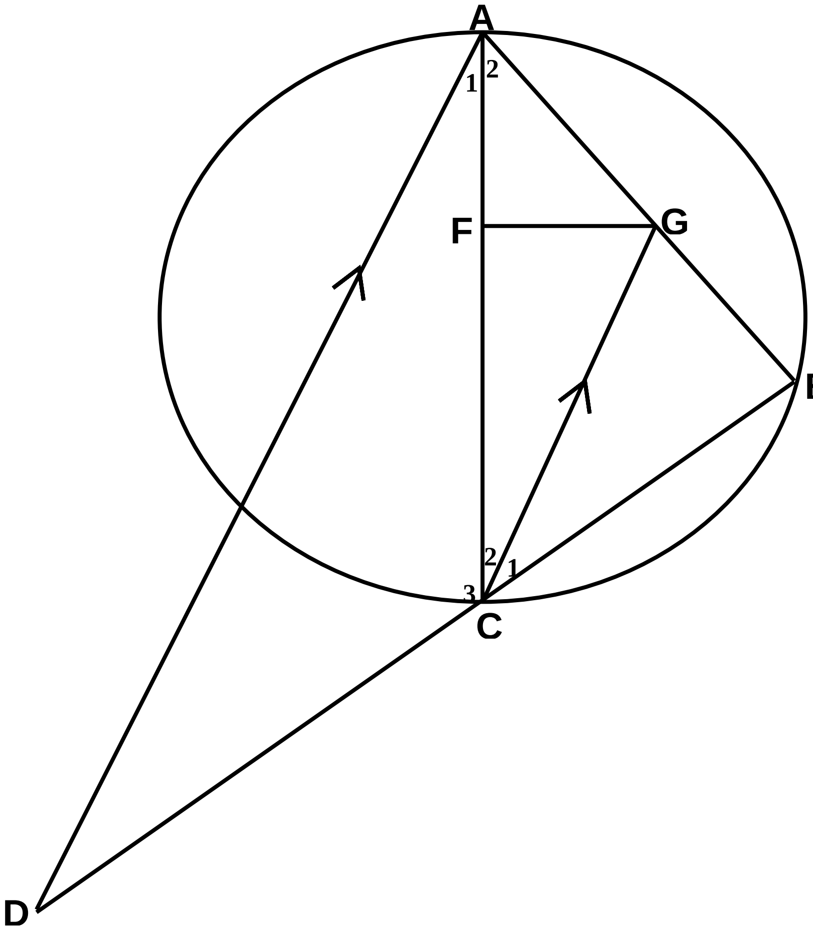
QUESTION 11.2



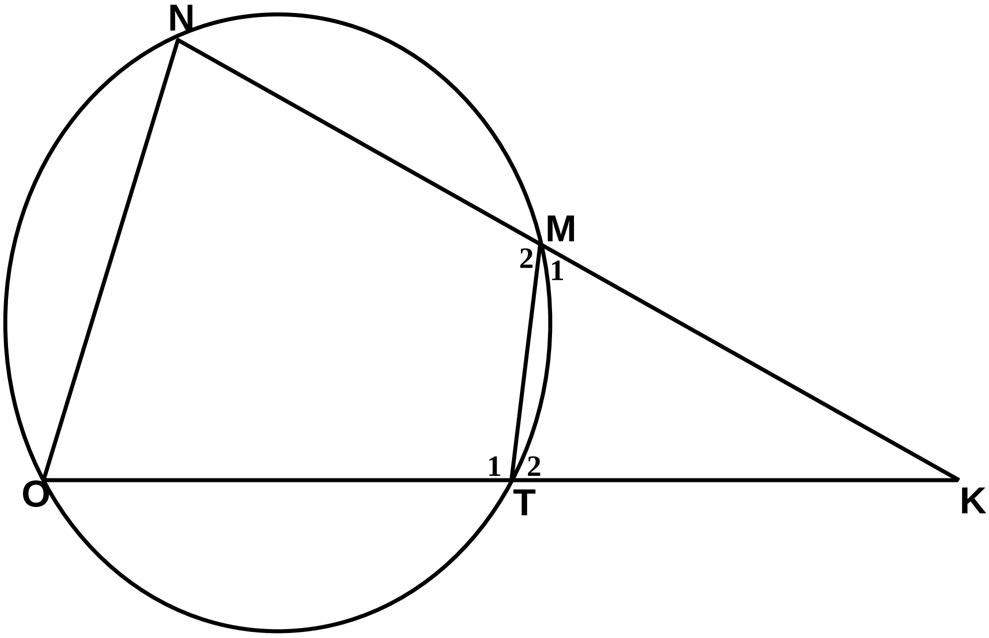
NAME:

DIAGRAM SHEET 6

QUESTION 12.2



QUESTION 12.3



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\,y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$