GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

PROOFREAD ☐ DATE __________
SIGNATURE ______________
PROOFREAD ☐ DATE __________
SIGNATURE ______________

MATHEMATICS
(First Paper)

TIME: 3 hours

MARKS: 150

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 13 questions.

2. Answer ALL the questions.

3. Show ALL calculations, diagrams, graphs, etc. that you have used in obtaining your answer.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable, non-graphic) may be used, unless stated otherwise.

6. Where necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are not necessarily drawn to scale.

8. A formula sheet is attached at the end of the question paper.

9. Number your answers exactly as the questions are numbered.

10. It is in your own interest to write neatly and legibly.

QUESTION 1

1.1 Calculate the values of x:

$x^2 + 2x = 0$

(3)

1.2 Hence, determine the positive values of x for which

$x^2 \geq -2x$. (3)

1.3 Solve for x:

1.3.1 $2x^2 - 3x - 7 = 0$ (correct to TWO decimal places). (3)

1.3.2 $2^0 + 2^{x-2} + 2^{x+1} + 2^x = 53$ (5)

[14]

QUESTION 2

2.1 Given:

$k = \sqrt{(x+1)^2 - 4}$, where k is a real number.

2.1.1 Solve for x if $k = 4$. (leave the answer in the simplest surd form). (5)

2.1.2 Write down the minimum value of k. (1)

2.2 Calculate the values of k, for which the equation
$3x^2 + 2x - k + 1 = 0$ has real roots. (4)

[10]

QUESTION 3

Given:

$$\sum_{p=1}^{n}(101-2p)$$

3.1 Write down the first THREE terms of the series. (3)

3.2 Calculate n for which $\sum_{p=1}^{n}(101-2p)=0$ (5)

[8]

QUESTION 4

4.1 The infinite number sequence below is a combination of two sequences:

1; 1; 1; 5; 1; 9; 1; 13; ...

Determine:

4.1.1 T_{31}, the thirty-first term of the sequence. (1)

4.1.2 S_{32}, the sum of the first thirty two terms of the sequence. (4)

4.1.3 the smallest value of the even number n for which $T_n > 97$. (3)

4.2 A plant is infected with bacteria. The number pattern below represents the number of bacterial cells in the plant at the end of every hour after infection.

3; 10; 19; 30 ...

4.2.1 Determine a general formula for the nth term (T_n) of this sequence. (4)

4.2.2 Calculate the number of bacterial cells in the plant at the end of the first day (24 hours). (2)

4.2.3 After the first day of infection, the plant undergoes treatment, which reduces the formation of new bacterial cells. The sequence below represents the number of new bacterial cells formed in the plant at the end of every day after infection:

670 ; 536 ; 428,8 ; …

The plant will perish if infected by more than 3 500 bacterial cells. If the treatment continues indefinitely, prove that the plant will survive this bacterial infection.

Show all calculations used in your answer. (3)

[17]

QUESTION 5

5.1 Convert an interest rate of 10% per annum, compounded monthly, to an annual interest rate, compounded semi-annually. (3)

5.2 Gavin purchases a house for R1 200 000. He pays a deposit of 10% of the value of the house. The bank grants him a loan for the outstanding amount, at an interest rate of 8,4% per annum compounded monthly, payable over a period of 20 years

5.2.1 Calculate the deposit Gavin pays on the house. (2)

5.2.2 Calculate Gavin's monthly repayments. (4)

5.2.3 Calculate the total amount Gavin would have paid for the house at the end of 20 years (3)

5.3 For two years, equipment worth R20 000, depreciates by 20% per annum according to the reducing-balance method. Each year thereafter, the annual depreciation is 2% less than that of the previous year. Determine the depreciated value of the equipment after 4 years. (4)

[16]

QUESTION 6

The function $f(x)=\dfrac{3}{x-1}-1$ is given.

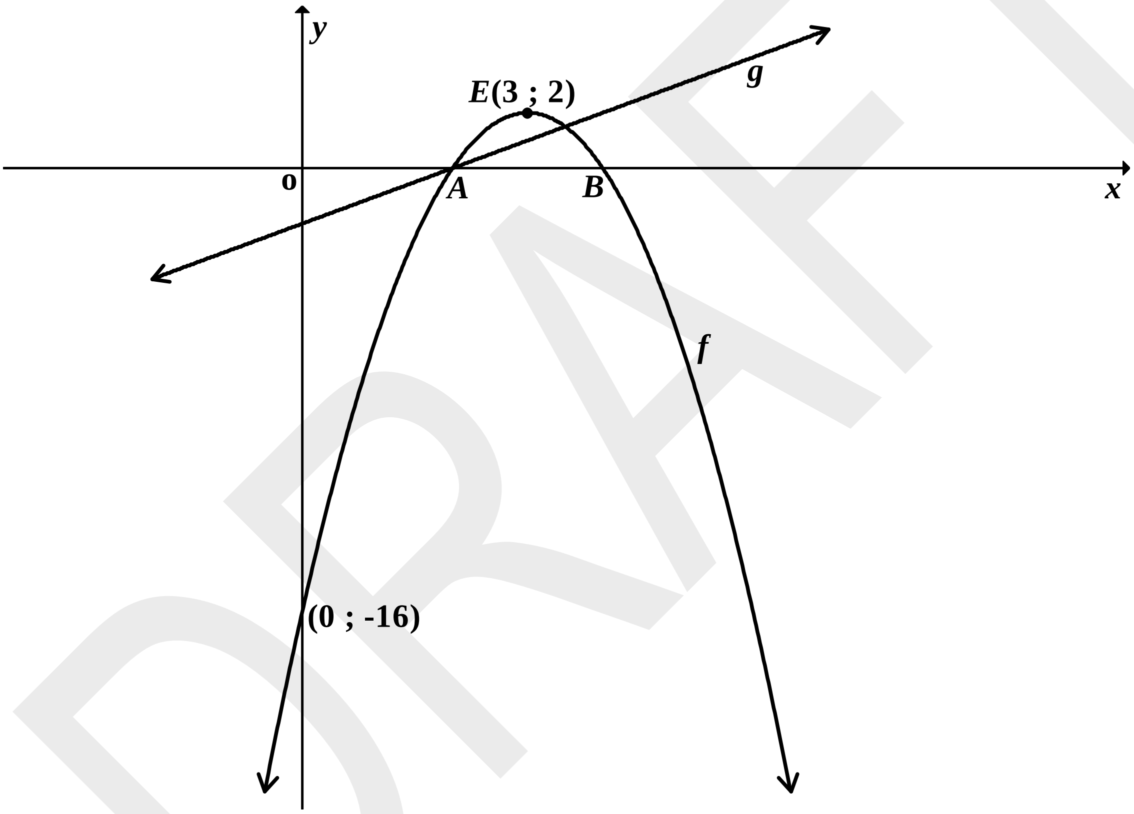
6.1 Draw a neat sketch graph of *f* in your answer book.
Clearly show all intercepts with the axes as well as all the asymptotes. (4)

6.2 Determine the equation(s) of the axis of symmetry of *f*. (2)

6.3 Determine the range of *f*. (1)
[7]

QUESTION 7

Sketched below are the graphs of $f(x) = a(x+p)^2 + q$ and $g(x) = mx + c$, where *a*, *p* ,*q*, *m* and *c* are real constants.
$E(3;2)$ is the turning point and $(0;-16)$ is the *y*-intercept of *f*.
The graph of *f* intersects the *x*-axis at *A* and *B*.
C, a point on the *y*-axis, is symmetrical to *A* about the line $y = -x$.
The graph of *g* passes through points *C* and *A*.



7.1 Show that the equation of *f* can be written as $f(x) = -2x^2 + 12x - 16$ (3)

7.2 Determine the coordinates of points *A*, *B* and *C*. (4)

7.3 Determine the equation of *g*. (3)

7.4 If $h(x) = \dfrac{-96}{x}$, calculate the point of intersection of *h* and *f*. (4)

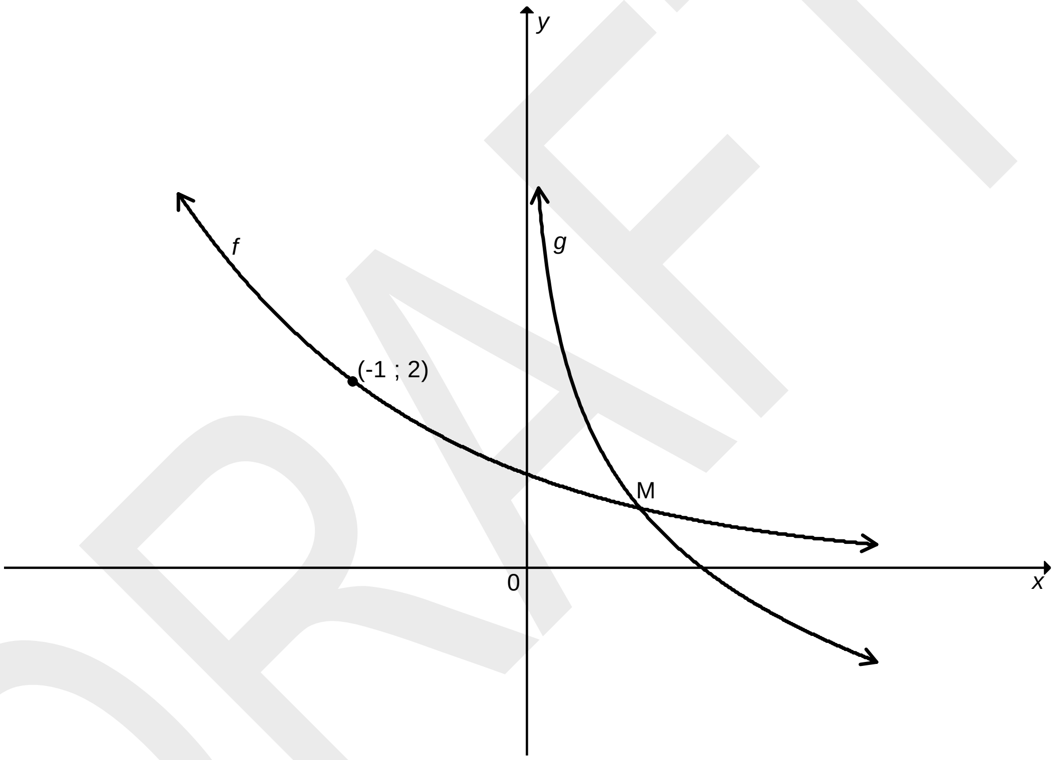
7.5 Write down the turning point of *p* , if $p(x) = -f\left(\dfrac{x}{2}\right)$. (2)

[16]

P.T.O.

QUESTION 8

The graphs of $f(x) = a^x$, $a > 0$ and $g(x) = f^{-1}(x)$, where g is the inverse of f, are sketched below: The point $(-1;2)$ lies on f. M is the point of intersection of f and g.



8.1 Show that $a = \frac{1}{2}$. (1)

8.2 Determine the equation of g in the form $g(x) = ...$ (2)

8.3 Write down the equation of the line which divides f and g symmetrically. (1)

8.4 If M is the point of intersection of f and the line $y = x$, show that
$x\log\frac{1}{2} - \log x = 0.$ (3)

[7]

QUESTION 9

9.1 Given $f(x) = 1 - 3x^2$.

9.1.1 Differentiate f from first principles (5)

9.1.2 Determine the equation of the tangent of f at $x = -2$. (3)

9.2 Evaluate:

9.2.1 $Dx\left[\left(1-x^{\frac{1}{2}}\right)\left(1-2x^{-\frac{1}{2}}\right)\right]$ (4)

9.2.2 $\frac{dy}{dp}$ if $y = \frac{p^3+1}{p+1}$ (3)

[15]

QUESTION 10

The displacement, (s) (distance from a fixed point), of a car moving towards a fixed point after t seconds, is given by

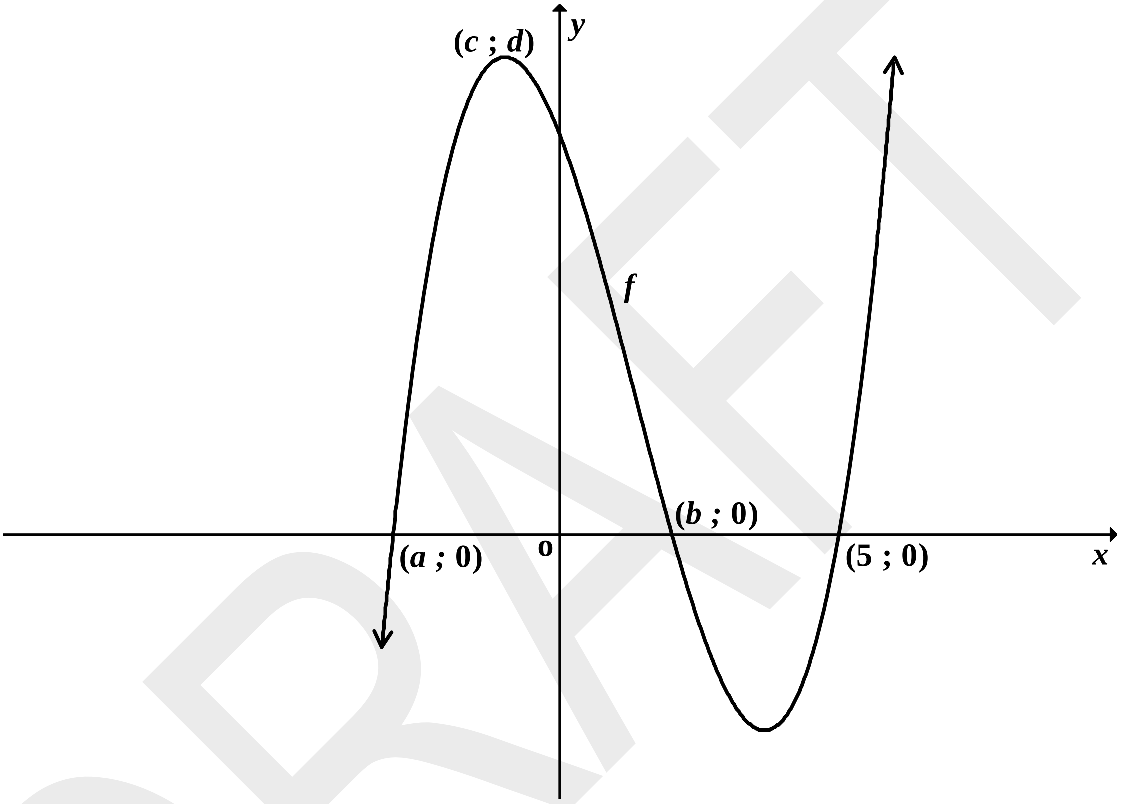
$s = t^3 - 9t^2 + 24t + 8$.

10.1 Determine an expression for the velocity (v) of the car.
(rate of change of distance with respect to time) (1)

10.2 Calculate the time t for which the car was reversing. (5)

[6]

QUESTION 11



Determine

11.1.1 the values of a and b. (4)

11.1.2 the values of c and d, where $(c; d)$ is a turning point of f. (4)

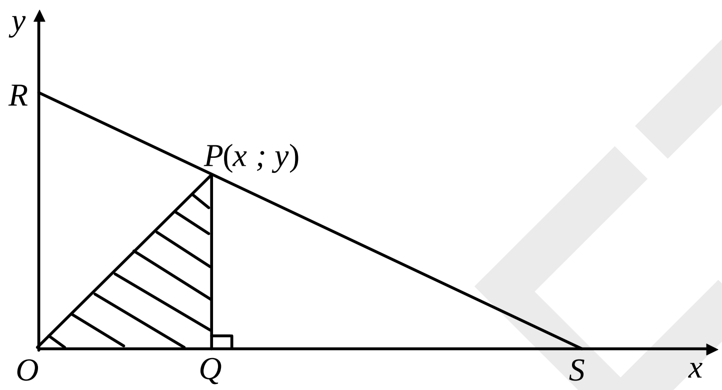
11.1.3 the values of x for which $f\,'(x) < 0$ and $f\,(x) \geq 0$. (2)

11.2 The turning points of a cubic graph $y = g(x)$ are $(-1; 6)$ and $(5; 4)$.
In your answer book, draw a sketch graph of y' (the derivative function of $y = g(x)$). Label only the x-intercepts clearly. (3)

[13]

QUESTION 12

In the diagram below, $P(x;y)$ is a point on line segment PS with equation $x+2y=18$, where $0 \le x \le 18$. Q is a point on the x-axis such that ΔPOQ forms a right-angled triangle. $PQ \perp OS$



12.1 Write the y-coordinate of point P in terms of x. (1)

12.2 Calculate the coordinates of P so that the area of ΔOPQ is at its maximum. (5)
[6]

QUESTION 13

13.1 Tebo writes an Art and a Music examination. He has 40% chance of passing the Music examination, 60% chance of passing the Art examination and 30% chance of passing both the Music and Art examination.

Calculate the probability that Tebo will pass the Music or Art examination. (3)

13.2 A survey was conducted asking 60 people with which hand they write and what colour hair they have. The results are summarized in the table below.

		HAND USED TO WRITE WITH		
		Right	Left	Total
	Light	***a***	***b***	**20**
	Dark	***c***	***d***	**40**
	Total	**48**	**12**	**60**

The survey concluded that the ‘hand used for writing’ and ‘hair colour’ are independent events.
Calculate the values of a, b, and c. (5)

13.3 The digits from 1 to 9 are used to make 5-digit codes.

13.3.1 Determine the number of 5-digit codes possible, if the digits are arranged in any order without repetition. (2)

13.3.2 Determine the number of 5-digit codes possible, if the code formed has to be an even number and the digits may not be repeated (3)

13.3.3 Determine the number of 5-digit codes possible, if the code formed only uses even digits and repetition of digits is allowed. (2)

[15]

TOTAL: 150

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END

5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$