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DEPARTMENT OF
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NATIONAL
SENIOR CERTIFICATE

GRADE 11

MATHEMATICS PAPER 2
MEMORANDUM
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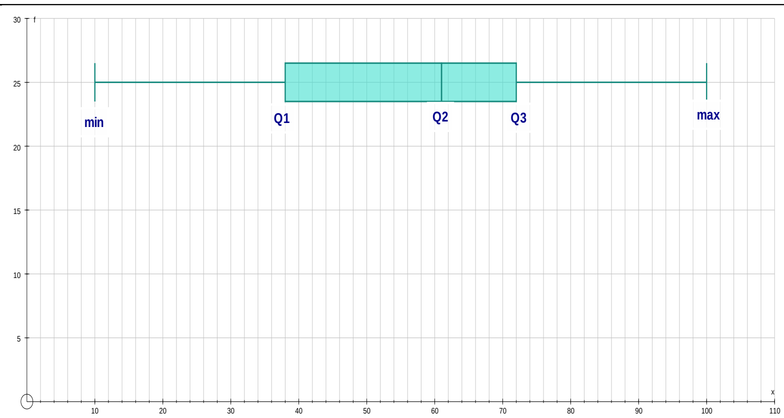
MARKS : 150

This solution paper consists of ten (10) pages, including the cover page.

QUESTION 1

1.1.1.	$\bar{x} = 28{,}86$	✓ mean	**(1)**
1.1.2.	$\sigma_x = 21{,}32$	✓ standard deviation(SD)	**(1)**
1.1.3.	$(\bar{x} \pm \sigma_x) = (28{,}86 \pm 21{,}32)$ $= (28{,}86 - 21{,}32; 28{,}86 + 21{,}32)$ $= (7{,}54\,; 50{,}18)$ $\therefore$ **10 athletes** will lie within one SD of the mean	✓ correct subst. into the formula ✓ interval ✓ answer	**(3)**
1.2.1.	$b = 20$ $IQR = 2a - a = 16$ $\therefore a = 16$ $\therefore d = 32$ $\bar{x} = \frac{127+c}{7} = 22$ $127 + c = 154$ $\therefore c = 27$	✓ $b = 20$ ✓ method ✓ $a = 16$ ✓ $d = 32$ ✓ method ✓ $c = 27$	**(6)**
			[11]

QUESTION 2

2.1.	100	✓ 100	**(1)**
2.2.	61	✓✓ Accept 61 or 62	**(2)**
2.3.		✓ $Q_1 = 38$ & $Q_3 = 72$ ✓ min $= 10$ & max $= 100$ ✓ median $= 61$ ✓✓ box and whisker diagram	**(5)**
2.4.	Skewed to the left	✓ answer	**(1)**
			[9]

QUESTION 3

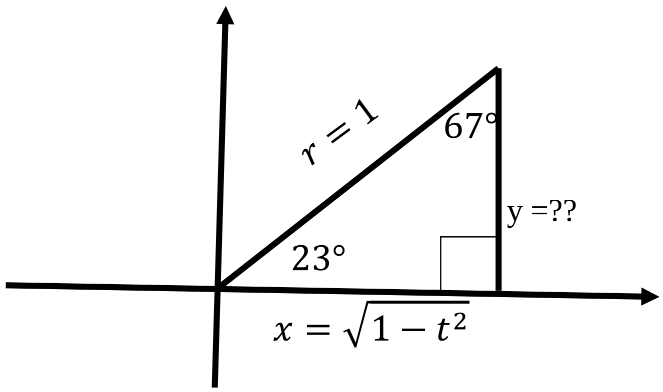
3.1.1.	$M(4;2)$	✓ x –value ✓ y –value	**(2)**
3.1.2.	$M_{AC} = \frac{6-(-2)}{2-6}$	✓ correct substitution	**(2)**

	$=\frac{8}{-4}$ $=\mathbf{-2}$	✓ answer	
3.1.3.	$y=mx+c$ $y=\frac{1}{2}x+c$ $2=\frac{1}{2}(4)+c$ $\therefore c=0$ $\therefore \boldsymbol{y}=\frac{1}{2}x$	✓ correct gradient ✓ **s**ubt. by M(4;2) ✓ equation	**(3)**
3.2.	$\boldsymbol{y}=\frac{1}{2}x$ L.H.S = 3 R.H.S = $\frac{1}{2}(6)=3$ L.H.S=R.H.S ∴ It does lie on the equation $\boldsymbol{y}=\frac{1}{2}x$	✓ L.H.S = 3 ✓ R.H.S = 3 ✓ Justification of the answer	**(3)**
3.3.	$y=mx+c$ $6=-2(2)+c$ $c=10$ $\therefore \boldsymbol{y}=-2x+10$	✓ subt. of m and point A or C ✓ equation	**(2)**
3.4.	$2y+\boldsymbol{x}=10$ $2(0)+\boldsymbol{x}=10$ $\therefore x=10$ $\therefore P(10;0)$	✓ subt. $y=0$ into the equation ✓ coordinate of point P	**(2)**
3.5.	$M(4;2)\ and\ B(-2;2)$ $\frac{x-2}{2}=4$ $x=10$ $\frac{y+2}{2}=2$ $y=2$ $\therefore \boldsymbol{D}(\mathbf{10}\,;\mathbf{2})$	✓ method ✓ $x-value$ ✓ $y-value$ ✓ coordinate of point D	**(4)**
			[18]

QUESTION 4

4.1.	$m_{MQ} = m_{NQ}$ $\frac{0-2}{t-0} = \frac{4-2}{3-0}$ $\frac{-2}{t} = \frac{2}{3}$ $2t = -6$ $t = -3$	✓ equating gradients ✓ correct subst. into gradients ✓ value of t	**(3)**
4.2.	$\tan\alpha = m_{NQ}$ $\tan\alpha = \frac{2}{3}$ $\alpha = 33{,}69°$ $\tan\beta = -\frac{1}{3}$ $\beta = 161{,}57°$ $\theta = \beta - \alpha$ (Ext. $\angle\ of\ a\ \Delta$) $\theta = 161{,}57° - 33{,}69°$ $\theta = 127{,}88°$	✓ $\tan\alpha = \frac{2}{3}$ ✓ $\alpha = 33{,}96°$ ✓ $\tan\beta = -\frac{1}{3}$ ✓ $\beta = 161{,}57°$ ✓ $\theta = 127{,}88°$	**(5)**
4.3.	$y = -\frac{1}{3}x + 6$ $0 = -\frac{1}{3}x + 6$ $(\ x - intercept\ at\ R)$ $-6 = -\frac{1}{3}x$ $x = 18$ $\therefore MR = 3 + 18$ $= 21$	✓ $x -$ intercept of SR ✓ method ✓ answer	**(3)**
4.4.	Area of $\Delta MNR = \frac{1}{2} b \times h$ $= \frac{1}{2}(21)(4)$ $= 42\ units^2$	✓ height y-value of N ✓ substitution ✓ answer	**(3)**
			[14]

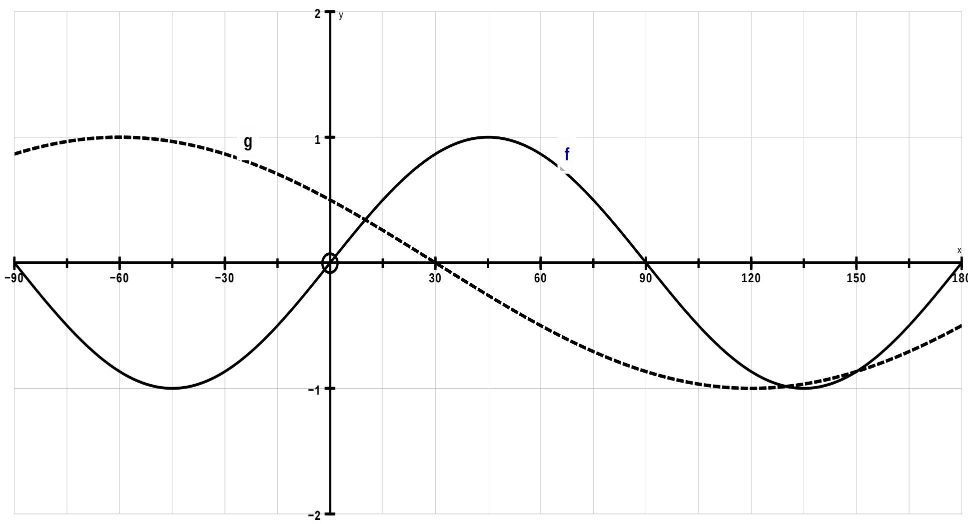
QUESTION 5

5.1.1.	$\cos 23° = \frac{\sqrt{1-t^2}}{1} = \frac{x}{r}$  $x^2 + y^2 = r^2$ (Theorem of Pythagorus) $(\sqrt{1-t^2})^2 + y^2 = (1)^2$ $y^2 = 1 - 1 + t^2$ $y = \pm\sqrt{t^2}$ $\therefore \boldsymbol{y = t}$ $\sin 23° = t$	✓ correct substitution into the equation ✓ $\boldsymbol{y = t}$ ✓ $\sin 23° = t$	**(3)**
5.1.2.	$\tan 113° = \tan(180° - 67°)$ $= -\tan 67°$ $= -\frac{\sqrt{1-t^2}}{t}$	✓ $\tan(180° - 67°)$ ✓ $-\tan 67°$ ✓ answer	**(3)**
5.1.3.	$\sin 67° = \sqrt{1-t^2}$	✓ answer	**(1)**
5.2.1.	$\frac{\tan(180°-x).\sin(90°+x)}{sin\ (-x)} - sin\ y.cos(90° - y)$ $= \frac{(-\tan x).(\cos x)}{-\sin x} - siny.\sin y$ $= \frac{-\frac{\sin x}{\cos x}.\cos x}{-\sin x} - sin^2 y$ $= 1 - sin^2 y$ $= cos^2 y$	✓ $-\boldsymbol{t}an\ x$ ✓ $\cos x$ ✓ $\sin y$ ✓ $-\sin x$ ✓ $\frac{\sin x}{\cos x}$ ✓ $1 - sin^2 y$ ✓ $cos^2 y$	**(7)**
5.2.2.	$\frac{cos^2(180° + 28°)}{tan\ (180° + 62°).cos\ 28°} \times \frac{1}{cos\ 30°.[-tan\ (180° - 60)].sin\ (180° + 28°)}$	✓ $-cos\ 28°$ ✓ $tan\ 62°$ ✓ $tan\ 60°$	**(9)**

	$=\frac{(-\cos 28°)^2}{\tan 62°.\cos 28°}\times\frac{1}{\cos 30°.\tan 60°.(-\sin 28°)}$ $=\frac{\cos^2 28°}{\tan(90°-28°).\cos 28°}\times\frac{1}{\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{1}(-\sin 28°)}$ $=\frac{\cos^2 28°}{\frac{1}{\tan 28°}.\cos 28°}\times-\frac{2}{3.\sin 28°}$ $=\frac{\cos^2 28°}{\frac{\cos 28°}{\sin 28°}\times\cos 28°}\times-\frac{2}{3.\sin 28°}$ $=\frac{\cos^2 28°}{1}\times\frac{\sin 28°}{\cos^2 28°}\times-\frac{2}{3.\sin 28°}$ $=-\frac{2}{3}$	✓ $-\sin 28°$ ✓ $\cos 30°=\frac{\sqrt{3}}{2}$ ✓ $\tan 60°=\sqrt{3}$ ✓ $\frac{1}{\tan 28°}$ ✓ $\frac{\cos 28°}{\sin 28°}$ ✓ $-\frac{2}{3}$	
5.3.	L.H.S $=\frac{\cos x}{1-\sin x}-\frac{\cos x}{1+\sin x}$ $=\frac{\cos x(1+\sin x)-\cos x(1-\sin x)}{(1-\sin x)(1+\sin x)}$ $=\frac{\cos x+\sin x.\cos x-\cos x+\sin x.\cos x}{1-\sin^2 x}$ $=\frac{2\sin x.\cos x}{\cos^2 x}$ $=\frac{2\sin x}{\cos x}$ $=2\tan x$ = R.H.S.	✓ L.C.D ✓ product of the numerator ✓ $1-\sin^2 x$ ✓ $2\sin x.\cos x$ ✓ $\cos^2 x$	**(5)**
			[28]

QUESTION 6

6.1.1.	$\sin 2x=\sin[90°-(x+60°)]$ $\sin 2x=\sin(30°-x)$ $2x=(30°-x)+k.360°, k\in Z$	 ✓ $(30°-x)$ ✓ $2x=(30°-x)$	**(6)**

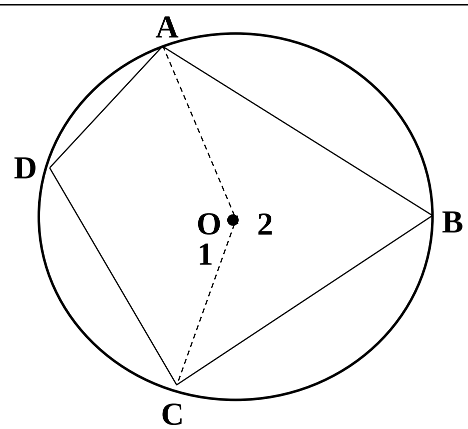
	$3x = 30° + k.360°, k \in Z$ $x = 10° + k.120°, k \in Z$ OR $2x = 180° - (30° - x) + k.360°, k \in Z$ $2x = 180° - 30° + x + k.360°, k \in Z$ $= 150° + k.360°, k \in Z$	✓ $x = 10°$ ✓ $+k.120°, k \in Z$ ✓$2x = 180° - (30° - x)$ ✓ $x = 150°$ $+k.360°, k \in Z$	
6.1.2.	$x = [10°; 130°; 150°]$	✓✓ answer	**(2)**
6.2.		✓ intercept with axes ✓ min & max points ✓ shape	**(3)**
6.3.1.	$x \in [(10°; 130°)U(150°; 180°)]$ or $10° \leq x \leq 130° and 150° \leq x \leq 180°$	✓✓ answer	**(2)**
6.3.2.	$x \in [(-90°; 0°)U(30°; 90°)]$ or $-90° \leq x \leq 0° and 30° \leq x \leq 90°$	✓✓ answer	**(2)**
			[15]

QUESTION 7

7.1.	$\hat{D}_1 + 60° + 120° - \alpha = 180°$ (sum of $\angle s$ of a Δ) $\hat{D}_1 = \alpha$	✓ Statement and reason ✓ answer	**(2)**
7.2.	In ΔBCD : $\frac{Sin\ D_1}{BC} = \frac{Sin\ 60°}{BD}$ (sine rule) $\frac{Sin\ \alpha}{4} = \frac{Sin\ 60°}{BD}$	✓substitution in sine rule	**(3)**

	$BD.Sin\ \alpha = 4.Sin\ 60°$ $BD = \frac{4(\frac{\sqrt{3}}{2})}{Sin\ \alpha}$ $BD = \frac{2\sqrt{3}}{Sin\ \alpha}$	✓ $\frac{\sqrt{3}}{2}$ ✓ answer	
7.3.	In ΔABD: $tan\ (90° - \theta) = \frac{BD}{AB}$ (ΔABD is right-angled) $\frac{1}{\tan\theta} = \frac{BD}{AB}$ $AB = BD.\tan\theta$ $AB = \frac{2\sqrt{3}.\tan\theta}{Sin\ \alpha}$	✓ ratio ✓ $\frac{1}{\tan\theta}$ ✓ substitution of BD	
			[8]

QUESTION 8

8.1.	 R.T.P. : $\hat{B} + \hat{D} = 180°$ Given : Circle O with points A, B, C and D on the circumference of the circle. Construction: Join OA and OC Proof : $\hat{O}_1 = 2\hat{B}$ ($\angle$ at the centre $=2 \times \angle$ on the circle) $\hat{O}_2 = 2\hat{D}$ ($\angle$ at the centre $=2 \times \angle$ on the circle) $\hat{O}_1 + \hat{O}_2 = 360°$ (revolution) $2\hat{B} + 2\hat{D} = 360°$ $2(\hat{B} + \hat{D}) = 360°$ $\therefore\ \hat{B} + \hat{D} = 180°$	✓Construction ✓Statement & reason ✓ Statement & reason ✓ Statement & reason ✓$2\hat{B} + 2\hat{D} = 360°$	**(5)**
8.2.1	$\hat{A} = 90°$ ($\angle$ subtended by a diameter)	✓statement ✓ reason	**(2)**

8.2.2.	$\hat{E}_1 = 80°$ (Opp. $\angle s$ of a cyclic quad.)	✓statement ✓ reason	**(2)**
8.2.3.	$\hat{E}_1 = \widehat{D}_1 + 35°$ (Ext. $\angle$ of a Δ= sum of 2. int. opp. $\angle s$) $80° = \widehat{D}_1 + 35°$ $\therefore \widehat{D}_1 = 45°$	✓ statement & reason ✓ answer	**(2)**
8.3.	$\hat{B}_1 + 90° + 55° = 180°$ (sum of $\angle s$ of a ΔABE) $\therefore \hat{B}_1 = 35°$ $\hat{F} = 35°$ (given) $\therefore \hat{B}_1 = \hat{F} = 35°$ AB$\mid\mid$CF (alternate $\angle s$ are =) OR $\hat{B}_1 + 90° + 55° = 180°$ (sum of $\angle s$ of a ΔABE) $\therefore \hat{B}_1 = 35°$ $\widehat{D}_1 = \hat{B}_2 = 45°$ (Ext. $\angle$ of a cyclic quad. = int. opp. $\angle$) $\hat{B}_1 + \hat{B}_2 + \hat{C} = 180°$ $\therefore$ AB$\mid\mid$CF (a pair of co-interior$\angle s$ is supplementary)	✓ statement & reason ✓ $\hat{B}_1 = 35°$ ✓ statement & reason ✓ Reason ✓ statement & reason ✓ $\hat{B}_1 = 35°$ ✓ statement & reason ✓ Reason	
			[15]

QUESTION 9

9.1.	$BC = 15\ cm$ (AOCD ⊥ BE ⇒ BC = CE)	✓statement ✓ reason	**(2)**
9.2.	$OC = 2a$	✓ answer	**(1)**
9.3.	$OB = OD$ (radii) $OB = OC + CD$ $OB = 2a + a$ $OB = 3a$	✓ statement & reason ✓ answer	**(2)**
9.4.	In ΔOBC: $OB^2 = BC^2 + OC^2$ (Theorem of Pythagorus) $(3a)^2 = (15)^2 + (2a)^2$ $5a^2 = 225$ $a^2 = 45$ $a = \sqrt{45}$ $a = 3\sqrt{5}$	✓ statement & reason ✓ substitution ✓ $a^2 = 45$ ✓ $a = 3\sqrt{5}$	**(4)**
			[9]

QUESTION 10

10.1.1	$\hat{C}_2 = \hat{A}_2$ ($\angle s$ subt. by = chords, i.e. AE = CD, given) $\hat{C}_4 = \hat{A}_2$ (tan-chord theorem) $\hat{C}_2 = \hat{C}_4$ (both = $\hat{A}_2$)	✓statement ✓ reason ✓ statement ✓reason	**(4)**
10.1.2	$\hat{C}_3 + \hat{C}_4 = \hat{A}_1 + \hat{A}_2$ (Ext. $\angle$ of a cyclic quad. = int. opp. $\angle$) $\hat{C}_4 = \hat{A}_2$ (tan-chord theorem) $\therefore \hat{C}_3 = \hat{A}_1$	✓statement ✓ reason ✓statement **&** reason	**(3)**

10.2.1	$\hat{U}_3 + \hat{U}_4 = 90°$ (∠ in a semi-circle)/(∠ subt. by a diameter) $\hat{X}_1 = 90°$ (Line from centre to the midpt. of chord) $\therefore \hat{U}_3 + \hat{U}_4 = \hat{X}_1 = 90°$ ∴ RU | | SY (a pair of corresponding $\angle s$ are =)	✓statement ✓ reason ✓statement ✓ reason ✓ reason	**(5)**
10.2.2	$\hat{R}_2 = y$ (tan-chord theorem) $= \hat{O}_1$ (alt. $\angle s$, RU | | SY) $\hat{T}_1 = \frac{1}{2}\hat{O}_1$ (∠ at the centre =2 × ∠ on the circle) $\therefore \hat{T}_1 = \frac{1}{2}y$ OR $\hat{R}_2 = y$ (tan-chord theorem) $\hat{R}_2 = \hat{O}_4$ (corresp. $\angle s$, RU | | SY) $\hat{T}_1 = \hat{S}_1$ ($\angle s$ opp. = sides i.e. OS = OT, radii) $\hat{O}_4 = \hat{T}_1 + \hat{S}_1$ (Ext. ∠ of a Δ = sum of 2. int. opp. $\angle s$) $\hat{O}_4 = 2\hat{T}_1$ $y = 2\hat{T}_1$ $\therefore \hat{T}_1 = \frac{1}{2}y$	✓ statement ✓ reason ✓ statement ✓reason ✓ statement ✓ reason ✓ statement ✓ reason ✓ statement ✓ reason ✓ statement ✓ reason	**(6)**
10.2.3	$\hat{U}_1 + \hat{U}_2 + \hat{U}_3 = 90°$ (OU(radius) ⊥ $UV(tangent)$) $\hat{T}_2 + \hat{T}_3 = 90°$ (OT(radius) ⊥ $TV(tangent)$) $\hat{U}_1 + \hat{U}_2 + \hat{U}_3 + \hat{T}_2 + \hat{T}_3 = 180°$ $TOUV$ is a cyclic quadrilateral (Opp. $\angle s$ of a quad. = 180°)	✓statement ✓ reason ✓statement ✓ reason ✓ reason	**(5)**
			[23]

[GRAND TOTAL = 150]